

The Time Value of Money

Learning Objectives

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|---|---|
| b¹ Explain the mechanics of compounding and bringing the value of money back to the present. | Compound Interest, Future, and Present Value |
| b² Understand annuities. | Annuities |
| b³ Determine the future or present value of a sum when there are nonannual compounding periods. | Making Interest Rates Comparable |
| b⁴ Determine the present value of an uneven stream of payments and understand perpetuities. | The Present Value of an Uneven Stream and Perpetuities |

In business, there is probably no other single concept with more power or applications than that of the time value of money. In his landmark book, *A History of Interest Rates*, Sidney Homer noted that if \$1,000 was invested for 400 years at 8 percent interest, it would grow to \$23 quadrillion—that would work out to approximately \$5 million per person on earth. He was not giving a plan to make the world rich but effectively pointing out the power of the time value of money.

The time value of money is certainly not a new concept. Benjamin Franklin had a good understanding of how it worked when he left £1,000 each to Boston and Philadelphia. With the gift, he left instructions that the cities were to lend the money, charging the going interest rate, to worthy apprentices. Then, after the money had been invested in this way for 100 years, they were to use a portion of the investment to build something of benefit to the city and hold some back for the future. Two hundred years later, Franklin's Boston gift resulted in the construction of the Franklin Union, has helped countless medical students with loans, and still has over \$3 million left in the account. Philadelphia, likewise, has reaped a significant reward from his gift with its portion of the gift growing to over \$2 million. Bear in mind that all this has come from a gift of £2,000 with some serious help from the time value of money.

The power of the time value of money can also be illustrated through a story Andrew Tobias tells in his book *Money Angles*. There he tells of a peasant who wins a chess tournament put on by the king. The king asks the peasant what he would like as the prize. The peasant answers that he would like for his village one piece of grain to be placed on the first square of his chessboard, two pieces of grain on the second square, four pieces on the third, eight on the fourth, and so forth. The king, thinking he was getting off easy, pledged on his word of honor that it would be done. Unfortunately for the king, by the time all 64 squares on the chessboard were filled, there were 18.5 million trillion grains of wheat on the board—the kernels were compounding at a rate of 100 percent over the 64 squares of the chessboard. Needless to say, no one in the village ever went hungry; in fact, that is so much wheat that if each kernel were one-quarter inch long (quite frankly, I have no idea how long a kernel of

wheat is, but Andrew Tobias's guess is one-quarter inch), if laid end to end, they could stretch to the sun and back 391,320 times.

Understanding the techniques of compounding and moving money through time is critical to almost every business decision. It will help you to understand such varied things as how stocks and bonds are valued, how to determine the value of a new project, how much you should save for your children's education, and how much your mortgage payments will be.



In the next six chapters, we focus on determining the value of the firm and the desirability of the investments it considers making. A key concept that underlies this material is the time value of money; that is, a dollar today is worth more than a dollar received a year from now. Intuitively this idea is easy to understand. We are all familiar with the concept of interest. This concept illustrates what economists call an opportunity cost of passing up the earning potential of a dollar today. This opportunity cost is the time value of money.

To evaluate and compare investment proposals, we need to examine how dollar values might accrue from accepting these proposals. To do this, all the dollar values must first be comparable. In other words, we must move all dollar flows back to the present or out to a common future date. An understanding of the time value of money is essential, therefore, to an understanding of financial management, whether basic or advanced.

Compound Interest, Future, and Present Value

We begin our study of the time value of money with some basic tools for visualizing the time pattern of cash flows. While timelines seem simple at first glance, they can be a tremendous help for more complicated problems.

Using Timelines to Visualize Cash Flows

As a first step in visualizing cash flows, we can construct a timeline, a linear representation of the timing of cash flows. A timeline identifies the timing and amount of a stream of cash flows—both cash received and cash spent—along with the interest rate it earns. Timelines are a critical first step used by financial analysts to solve financial problems. We will refer to them often throughout this text.

To illustrate how to construct a timeline, consider the following example, where we receive annual cash flows over the course of 4 years. The following timeline shows our cash inflows and outflows from time period 0 (the present) until the end of year 4:

	$r = 10\%$				
YEARS	0	1	2	3	4
Cash Flow	-100	30	20	-10	50
	↑		↑		↑
	Today & Beginning of Period 1		End of Period 2 & Beginning of Period 3		End of Period 4 & Beginning of Period 5



1 Explain the mechanics of compounding and bringing the value of money back to the present.

For our purposes, time periods are identified on the top of the timeline, and in this example, the time periods are measured in years, which are indicated on the far left of the timeline. Thus, time period 0 is both today and the beginning of the first year. The dollar amount of the cash flow received or spent at each time period is shown below the timeline. Positive values represent *cash inflows*. Negative values represent *cash outflows*. For example, in the timeline shown, a \$100 cash outflow occurs today, or at time 0, followed by cash inflows of \$30 and \$20 at the end of years 1 and 2, a negative cash flow (a cash outflow) of \$10 at the end of year 3, and finally a cash inflow of \$50 at the end of year 4.

The units of measurement on the timeline are time periods and are typically expressed in years but could be expressed as months, days, or any unit of time. However, for now, let's assume we're looking at cash flows that occur annually. Thus, the distance between 0 and 1 represents the period between today and the end of the first year. Consequently, time period 0 indicates today, while time period 1 represents the end of the first year, which is also the beginning of the second year. (You can think of it as being both the last second of the first year and the first second of the second year.) The interest rate, which is 10 percent in this example, is listed above the timeline.

In this chapter and throughout the text, we will often refer to the idea of moving money through time. Because this concept is probably not familiar to everyone, we should take a moment to explain it. Most business investments involve investing money today. Then, in subsequent years, the investment produces cash that comes back to the business. To evaluate an investment, you need to compare the amount of money the investment requires today with the amount of money the investment will return to you in the future and adjust these numbers for the fact that a dollar today is worth more than a dollar in the future. Timelines simplify solving time value of money problems, and they are not just for beginners; experienced financial analysts use them as well. In fact, the more complex the problem you're trying to solve, the more valuable a timeline will be in terms of helping you visualize exactly what needs to be done.

Most of us encounter the concept of compound interest at an early age. Anyone who has ever had a savings account or purchased a government savings bond has received compound interest. **Compound interest** occurs when *interest paid on the investment during the first period is added to the principal; then, during the second period, interest is earned on this new sum*.

For example, suppose we place \$100 in a savings account that pays 6 percent interest, compounded annually. How will our savings grow? At the end of the first year we have earned 6 percent, or \$6 on our initial deposit of \$100, giving us a total of \$106 in our savings account, thus,

$$\begin{aligned}\text{Value at the end of year 1} &= \text{present value} \times (1 + \text{interest rate}) \\ &= \$100(1 + 0.06) \\ &= \$100(1.06) \\ &= \$106\end{aligned}$$

Carrying these calculations one period further, we find that we now earn the 6 percent interest on a principal of \$106, which means we earn \$6.36 in interest during the second year.

Why do we earn more interest during the second year than we did during the first? Simply because we now earn interest on the sum of the original principal and the interest we earned in the first year. In effect we are now earning interest on interest; this is the concept of compound interest. Examining the mathematical formula illustrating the earning of interest in the second year, we find

$$\text{Value at the end of year 2} = \text{value at the end of year 1} \times (1 + r)$$

where r = the annual interest (or discount) rate, which for our example gives

$$\begin{aligned}\text{Value at the end of year 2} &= \$106 \times (1.06) \\ &= \$112.36\end{aligned}$$

compound interest the situation in which interest paid on an investment during the first period is added to the principal. During the second period, interest is earned on the original principal plus the interest earned during the first period.



REMEMBER YOUR PRINCIPLES

In this chapter we develop the tools to incorporate **Principle 2: Money Has a Time Value** into our calculations. In coming chapters we use this concept to measure value by bringing the benefits and costs from a project back to the present.

Looking back, we can see that the **future value** at the end of year 1, or \$106, is actually equal to the present value times $(1 + r)$, or $\$100(1 + 0.06)$. Moving forward to year 2 we find,

future value the amount to which your investment will grow, or a future dollar amount.

$$\begin{aligned}\text{Value at the end of year 2} &= \text{present value} \times (1 + r) \times (1 + r) \\ &= \text{present value} \times (1 + r)^2\end{aligned}$$

Carrying this forward into the third year, we find that we enter the year with \$112.36 and we earn 6 percent, or \$6.74 in interest, giving us a total of \$119.10 in our savings account. This can be expressed as

$$\begin{aligned}\text{Value at the end of year 3} &= \text{present value} \times (1 + r) \times (1 + r) \times (1 + r) \\ &= \text{present value} \times (1 + r)^3\end{aligned}$$

By now a pattern is becoming apparent. We can generalize this formula to illustrate the future value of our investment if it is compounded annually at a rate of r for n years to be

$$\text{Future value} = \text{present value} \times (1 + r)^n$$

Letting FV_n stand for the future value at the end of n periods and PV stand for the present value, we can rewrite this equation as

$$FV_n = PV(1 + r)^n \quad (5-1)$$

We also refer to $(1 + r)^n$ as the **future value factor**. Thus, to find the future value of a dollar amount, all you need to do is multiply that dollar amount times the appropriate future value factor,

future value factor the value of $(1 + r)^n$ used as a multiple to calculate an amount's future value.

$$\text{Future value} = \text{present value} \times (\text{future value factor})$$

where future value factor $= (1 + r)^n$.

Figure 5-1 illustrates how this investment of \$100 would continue to grow for the first 20 years at a compound interest rate of 6 percent. Notice how the amount of interest earned annually increases each year. Again, the reason is that each year interest is

FIGURE 5-1 \$100 Compounded at 6 Percent Over 20 Years

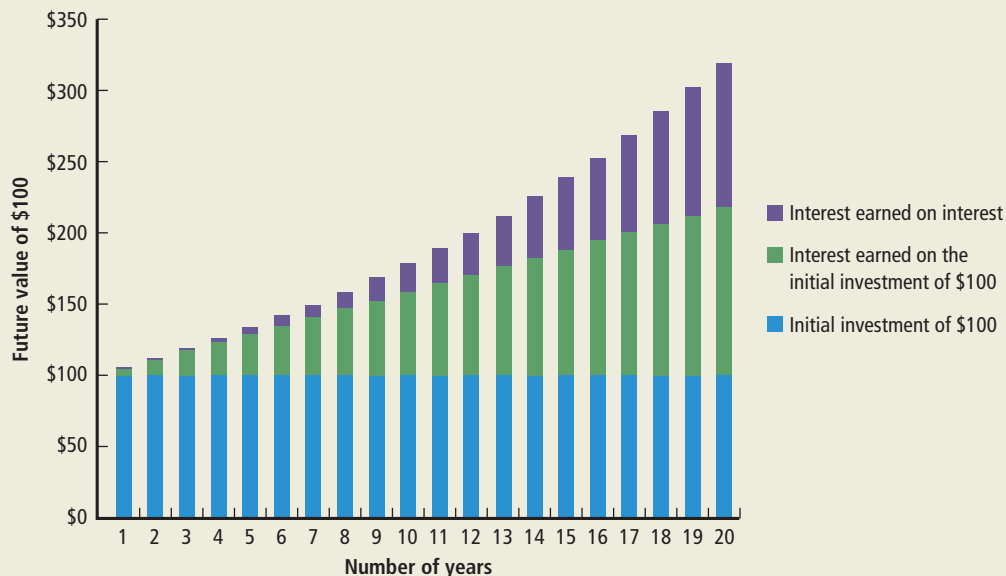
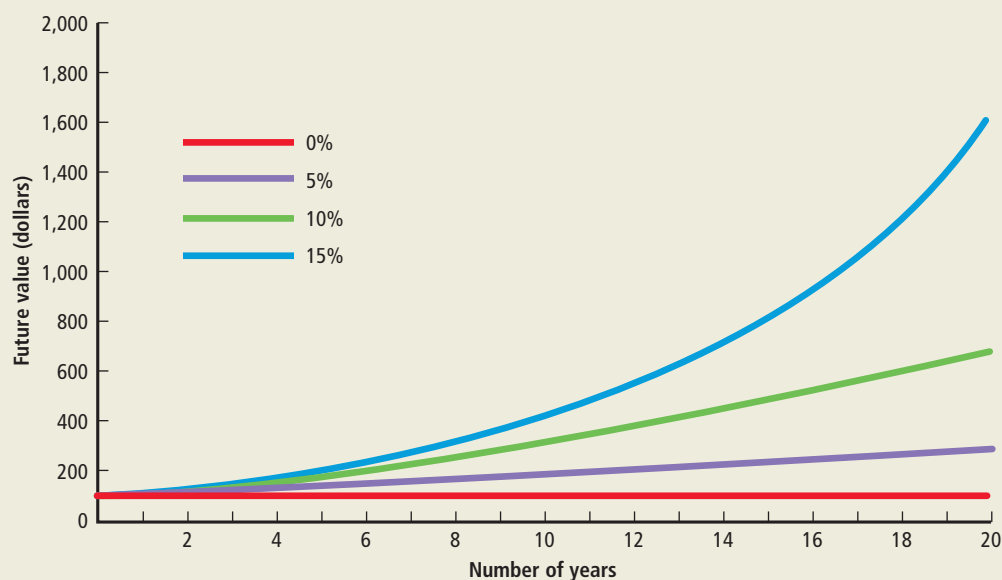


FIGURE 5-2 The Future Value of \$100 Initially Deposited and Compounded at 0, 5, 10, and 15 Percent

simple interest if you only earned interest on your initial investment, it would be referred to as simple interest.

received on the sum of the original investment plus any interest earned in the past. The situation in which interest is earned on interest that was earned in the past is referred to as compound interest. *If you only earned interest on your initial investment*, it would be referred to as **simple interest**.

When we examine the relationship between the number of years an initial investment is compounded for and its future value, as shown graphically in Figure 5-2, we see that we can increase the future value of an investment by either increasing the number of years for which we let it compound or by compounding it at a higher interest rate. We can also see this from equation (5-1) because an increase in either r or n while the present value is held constant results in an increase in the future value.

EXAMPLE 5.1

Calculating the future value of an investment

If we place \$1,000 in a savings account paying 5 percent interest compounded annually, how much will our account accrue to in 10 years?

STEP 1: FORMULATE A SOLUTION STRATEGY

The future value of the savings account takes into account the present value of the account and multiplies it by the annual interest rate each year it is applied, that is,

$$\text{Future value} = \text{present value} \times (1 + r)^n \quad (5-1)$$

where r = annual interest rate and n = the number of years.

STEP 2: CRUNCH THE NUMBERS

Substituting present value, percent, and 10 years into equation (5-1), we get the following:

$$\begin{aligned} FV_n &= \$1,000(1 + 0.05)^{10} \\ &= \$1,000(1.62889) \\ &= \$1,628.89 \end{aligned} \quad (5-1)$$

STEP 3: ANALYZE YOUR RESULTS

At the end of the 10 years, we will have \$1,628.89 in our savings account.

Techniques for Moving Money Through Time

There are three different approaches that you can use to solve a time value of money problem. The first is to simply do the math. Financial calculators are a second alternative, and there are a number of them that do a good job of solving time value of money problems. Based on many years of experience, the Texas Instruments BA II Plus or the Hewlett-Packard 10BII calculators would be excellent choices. Finally, spreadsheets can move money through time, and in the real world, they are without question the tool of choice. Now let's take a look at all three alternatives.

Mathematical Calculations If we want to calculate the future value of an amount of money, the mathematical calculations are relatively straightforward. However, as we will see, time value of money calculations are easier using a financial calculator or spreadsheet. Those are the chosen methods in the real world.

EXAMPLE 5.2

Calculating future value of an investment

If we invest \$500 in a bank where it will earn 8 percent interest compounded annually, how much will it be worth at the end of 7 years?

STEP 1: FORMULATE A SOLUTION STRATEGY

The future value of the savings account is found using the same method as in Example 5.1 using equation (5-1) as follows:

$$\text{Future value} = \text{present value} \times (1 + r)^n \quad (5-1)$$

STEP 2: CRUNCH THE NUMBERS

Substituting into equation (5-1) we compute the future value of our account as follows:

$$FV = \$500(1 + 0.08)^7 = \$856.91$$

STEP 3: ANALYZE YOUR RESULTS

At the end of 7 years, we will have \$856.91 in our savings account. In the future we will find several uses for equation (5-1); not only can we find the future value of an investment but we can also solve for the present value, r , or n .

Using a Financial Calculator Before we review the use of the financial calculator, let's review the five "time value of money" keys on a financial calculator. Although the specific keystrokes used to perform time value of money calculations differ slightly when using financial calculators made by different companies, the symbols used are basically the same. Below are the keystrokes as they appear on a Texas Instruments BA II Plus calculator.

MENU KEY	DESCRIPTION
Menu Keys we will use in this chapter include:	
N	Stores (or calculates) the total number of payments or compounding periods.
I/Y	Stores (or calculates) the interest (or discount or growth) rate per period.
PV	Stores (or calculates) the present value of a cash flow (or series of cash flows).
FV	Stores (or calculates) the future value, that is, the dollar amount of a final cash flow (or the compound value of a single flow or series of cash flows).
PMT	Stores (or calculates) the dollar amount of each annuity (or equal) payment deposited or received.

The keys shown here correspond to the TI BA II Plus calculator. Other financial calculators have essentially the same keys. They are simply labeled a little differently. We should stop and point out that the labels on the keys of financial calculators are slightly different

than what we have been using in our mathematical formulas. For example, we have used a lowercase n to refer to the number of compounding periods, whereas an uppercase N appears on the Texas Instruments BA II Plus calculator. Likewise, the I/Y key refers to the rate of interest per period, whereas we have used r . Some financial calculators also have a CPT key, which stands for “compute.” It is simply the key you press when you want the calculator to begin solving your problem. Finally, the PMT key refers to a fixed payment received or paid each period.

At this point you might be wondering exactly why we are using different symbols in this book than are used on calculators. The answer is that, unfortunately, the symbols used by the different companies that design and make financial calculators are not consistent. The symbols in Microsoft Excel are somewhat different as well.

An important thing to remember when using a financial calculator is that cash outflows (investments you make rather than receive) generally have to be entered as negative numbers. In effect, a financial calculator sees money as “leaving your hands” and therefore taking on a *negative* sign when you invest it. The calculator then sees the money “returning to your hands” and therefore taking on a *positive* sign after you’ve earned interest on it. Also, every calculator operates a bit differently with respect to entering variables. Needless to say, it is a good idea to familiarize yourself with exactly how your calculator functions.¹

Once you’ve entered all the variables you know, including entering a zero for any variable with a value of zero, you can let the calculator do the math for you. If you own a Texas Instruments BA II Plus calculator, press the compute key (CPT), followed by the key corresponding to the unknown variable you’re trying to determine. With a Hewlett-Packard calculator, once the known variables are entered, you need only press the key corresponding to the final variable to calculate its value.

A good starting place when working a problem is to write down all the variables you know. Then assign a value of zero to any variables that are not included in the problem. Once again, make sure that the sign you give to the different variables reflects the direction of the cash flow.

Calculator Tips—Getting It Right Calculators are pretty easy to use. But there are some common mistakes that are often made. So, before you take a crack at solving a problem using a financial calculator ensure that you take the following steps:

1. **Set your calculator to one payment per year.** Some financial calculators use monthly payments as the default. Change it to annual payments. Then consider n to be the number of periods and r the interest rate per period.
2. **Set your calculator to display at least four decimal places or to floating decimal place (nine decimal places).** Most calculators are preset to display only two decimal places. Because interest rates are so small, change your decimal setting to at least four.
3. **Set your calculator to the “end” mode.** Your calculator will assume cash flows occur at the end of each time period.

When you’re ready to work a problem, remember:

1. **Every problem must have at least one positive and one negative number.** The pre-programming within the calculator assumes that you are analyzing problems in which money goes out (outflows) and comes in (inflows), so you have to be sure to enter the sign appropriately. If you are using a BA II Plus calculator and get an “Error 5” message, that means you are solving for either r or n and you input both the present and future values as positive numbers—correct that and re-solve the problem.
2. **You must enter a zero for any variable that isn’t used in a problem, or you have to clear the calculator before beginning a new problem.** If you don’t clear your calculator or enter a value for one of the variables, your calculator won’t assume that that variable is zero. Instead, your calculator will assume it carries the same number as

¹Appendix A at www.pearsonhighered.com/keown provides a tutorial that covers both the Texas Instruments BA II Plus and the Hewlett-Packard 10BII calculators.

it did during the previous problem. So be sure to clear out the memory (CLR, TVM) or enter zeros for variables not included in the problem.

3. **Enter the interest rate as a percent, not a decimal.** That means 10 percent must be entered as 10 rather than 0.10.

Two popular financial calculators are the Texas Instruments BA II Plus (TI BA II Plus) and the Hewlett-Packard 10BII (HP 10BII). If you're using one of these calculators and encounter any problems, take a look at Appendix A at www.pearsonhighered.com/keown. It provides a tutorial for both of these calculators.

Spreadsheets Practically speaking, most time value of money calculations are now done on computers with the help of spreadsheet software. Although there are competing spreadsheet programs, the most popular is Microsoft Corporation's Excel®. Just like financial calculators, Excel has "built-in" functions that make it really easy to do future value calculations.

Excel Tips—Getting It Right

1. **Take advantage of the formula help that Excel offers.** All Excel functions are set up the same way: First, with the "=" sign; second, with the function's name (for example, FV or PV); and third, with the inputs into that function enclosed in parentheses. The following, for example, is how the future value function looks:

= FV(rate,nper,pmt,pv)

When you begin typing in your Excel function in a spreadsheet cell (that is, a box where you enter a single piece of data), all the input variables will appear at the top of the spreadsheet in their proper order, so you will know what variable must be entered next. For example, = FV(rate,nper,pmt,pv,type) will come into view, with **rate** appearing in bold because rate is the next variable to enter. This means you don't really need to memorize the functions because they will appear when you begin entering them.

2. **If you're lost, click on "Help."** On the top row of your Excel spreadsheet you'll notice the word *Help* listed—another way to get to the help link is to hit the F1 button. When you're lost, the help link is your friend. Click on it and enter "PV" or "FV" in the search bar, and the program will explain how to calculate each. All of the other financial calculations you might want to find can be found the same way.

3. **Be careful about rounding the r variable.** For example, suppose you're dealing with the interest rate 6.99 percent compounded monthly. This means you will need to enter the interest rate per month, which is = 6.99%/12, and since you are performing division in the cell, you need to put an "=" sign before the division is performed. Don't round the result of 0.0699/12 to 0.58 and enter 0.58 as r . Instead, enter = 6.99/12 or as a decimal = 0.0699/12 for r .

Also, you'll notice that while the inputs using an Excel spreadsheet are almost identical to those on a financial calculator, the interest rate in Excel is entered as *either* a decimal (0.06) *or* a whole number followed by a % sign (6%) rather than a 6 (as you would enter if you were using a financial calculator).

4. **Don't let the Excel notation fool you.** Excel doesn't use r or I/Y to note the interest rate. Instead, it uses the term *rate*. Don't let this bother you. All of these notations refer to the same thing—the interest rate. Similarly, Excel doesn't use n to denote the number of periods. Instead, it uses *nper*. Once again, don't let this bother you. Both n and *nper* refer to the number of periods.
5. **Don't be thrown off by the "Type" input variable.** You'll notice that Excel asks for a new variable that we haven't talked about yet, "type." Again, if you type "= FV" in a cell, "= FV(rate,nper,pmt,pv,type)" will immediately appear just below that cell. Don't let this new variable, "type," throw you off. The variable "type" refers to whether the payments, *pmt*, occur at the end of each period (which is considered type = 0) or the beginning of each period (which is considered type = 1). But you don't have to worry about it at all because the default on "type" is 0. *Thus, if you don't enter a value for "type," it will assume that the payments occur at the end of each period.* We're going to assume they

occur at the end of each period, and if they don't, we'll deal with them another way that will be introduced later in this chapter. You'll also notice that since we assume that all payments occur at the end of each period unless otherwise stated, we ignore the "type" variable.

Some of the more important Excel functions that we will be using throughout the book include the following (again, we are ignoring the "type" variable because we are assuming cash flows occur at the end of the period):

CALCULATION (WHAT IS BEING SOLVED FOR)	FORMULA
Present value	=PV(rate, nper, pmt, fv)
Future value	=FV(rate, nper, pmt, pv)
Payment	=PMT(rate, nper, pv, fv)
Number of periods	=NPER(rate, pmt, pv, fv)
Interest rate	=RATE(nper, pmt, pv, fv)

Reminders: First, just as in using a financial calculator, the outflows have to be entered as negative numbers. In general, each problem will have two cash flows—one positive and one negative. Second, a small, but important, difference between a spreadsheet and a financial calculator: When using a financial calculator, you enter the interest rate as a percent. For example, 6.5 percent is entered as 6.5. However, with the spreadsheet, the interest rate is entered as a decimal, thus 6.5 percent would be entered as 0.065 or, alternatively, as 6.5 followed by a % sign.

EXAMPLE 5.3

Calculating the future value of an investment

If you put \$1,000 into an investment paying 20 percent interest compounded annually, how much will your account grow to in 10 years?

STEP 1: FORMULATE A SOLUTION STRATEGY

Let's start with a timeline to help you visualize the problem:

	$r = 20\%$										
YEARS	0	1	2	3	4	5	6	7	8	9	10
Cash Flows	-1,000										
											Future Value = ?

The future value of our savings account can be computed using equation (5-1) as follows:

$$\text{Future value} = \text{present value} \times (1 + r)^n \quad (5-1)$$

STEP 2: CRUNCH THE NUMBERS

Using the Mathematical Formulas

Substituting present value = \$1,000, $r = 20$ percent, and $n = 10$ years into equation (5-1), we get

$$\begin{aligned}
 \text{Future value} &= \text{present value} \times (1 + r)^n && (5-1) \\
 &= \$1,000(1 + 0.20)^{10} \\
 &= \$1,000(6.19174) \\
 &= \$6,191.74
 \end{aligned}$$

Thus, at the end of 10 years, you will have \$6,191.74 in your investment. In this problem we've invested \$1,000 at 20 percent and found that it will grow to \$6,191.74 after 10 years. These are actually equivalent values expressed in terms of dollars from different time periods where we've assumed a 20 percent compound rate.

Using a Financial Calculator

A financial calculator makes this even simpler. If you are not familiar with the use of a financial calculator, or if you have any problems with these calculations, check out the tutorial on financial calculators in Appendix A at www.pearsonhighered.com/keown. There you'll find an introduction to financial calculators and the time value of money along with calculator tips to make sure that you come up with the right answers.

Enter	10	20	− 1,000	0	
	N	I/N	PV	PMT	FV
Solve for					6,192

Notice that you input the present value with a negative sign. In effect, a financial calculator sees money as “leaving your hands” and therefore taking on a negative sign when you invest it. In this case you are investing \$1,000 right now, so it takes on a negative sign—as a result, the answer takes on a positive sign.

Using an Excel Spreadsheet

You'll notice the inputs using an Excel spreadsheet are almost identical to those on a financial calculator. The only difference is that the interest rate in Excel is entered as *either* a decimal (0.20) *or* a whole number followed by a % sign (20%) rather than as 20 (as you would enter if you were using a financial calculator). Again, the present value should be entered with a negative value so that the answer takes on a positive sign.

	A	B
1	interest rate (rate) =	20.00%
2	number of periods (nper) =	10
3	payment (pmt) =	\$0
4	present value (pv) =	(\$1,000)
5		
6	future value (fv) =	\$6,192
7		
8	Excel formula =FV(rate,nper,pmt,pv)	
9	Entered in cell b6: =FV(b1,b2,b3,b4)	

STEP 3: ANALYZE YOUR RESULTS

Thus, at the end of 10 years, you will have \$6,192 in your investment. In this problem we've invested \$1,000 at 20 percent and found that it will grow to \$6,192 after 10 years. These are actually equivalent values expressed in terms of dollars from different time periods where we've assumed a 20 percent compound rate.

Two Additional Types of Time Value of Money Problems

Sometimes the time value of money does not involve determining either the present value or future value of a sum, but instead deals with either the number of periods in the future, n , or the rate of interest, r . For example, to answer the following question you will need to calculate the value for n .

- ◆ How many years will it be before the money I have saved will be enough to buy a second home?

Similarly, questions such as the following must be answered by solving for the interest rate, r .

- ◆ What rate do I need to earn on my investment to have enough money for my newborn child's college education ($n = 18$ years)?
- ◆ What interest rate has my investment earned?

Fortunately, with the help of a financial calculator or an Excel spreadsheet, you can easily solve for r or n in any of the above situations. It can also be done using the mathematical formulas, but it's much easier with a calculator or spreadsheet, so we'll stick to them.

Solving for the Number of Periods Suppose you want to know how many years it will take for an investment of \$9,330 to grow to \$20,000 if it's invested at 10 percent annually. Let's take a look at solving this using a financial calculator and an Excel spreadsheet.

Using a Financial Calculator With a financial calculator, all you do is substitute in the values for I/Y, PV, and FV and solve for N:

Enter		10	− 9,330	0	20,000
	N	I/Y	PV	PMT	FV
Solve for	8				

You'll notice that PV is input with a negative sign. In effect, the financial calculator is programmed to assume that the \$9,330 is a cash outflow, whereas the \$20,000 is money that you receive. If you don't give one of these values a negative sign, you can't solve the problem.

Using an Excel Spreadsheet With Excel, solving for n is straightforward. You simply use the = NPER function and input values for rate, pmt, pv, and fv.

	A	B
1	interest rate (rate) =	10.00%
2	payment (pmt) =	0
3	present value (pv) =	(\$9,330)
4	future value (fv) =	\$20,000
5		
6	number of periods (nper) =	8
7		
8	Excel formula =nper(rate,pmt,pv,fv)	
9	Entered in cell b6: =nper(b1,b2,b3,b4)	

Solving for the Rate of Interest You have just inherited \$34,946 and want to use it to fund your retirement in 30 years. If you have estimated that you will need \$800,000 to fund your retirement, what rate of interest would you have to earn on your \$34,946 investment? Let's take a look at solving this using a financial calculator and an Excel spreadsheet to calculate the interest rate.

Using a Financial Calculator With a financial calculator, all you do is substitute in the values for N, PV, and FV, and solve for I/Y:

Enter	30	− 34,946	0	800,000
	N	I/Y	PV	PMT
Solve for	11			FV

Using an Excel Spreadsheet With Excel, the problem is also very easy. You simply use the = RATE function and input values for nper, pmt, pv, and fv.

	A	B
1	number of periods (nper) =	30
2	payment (pmt) =	0
3	present value (pv) =	(\$34,946)
4	future value (fv) =	\$800,000
5		
6	interest rate (rate) =	11.00%
7		
8	Excel formula =rate(nper,pmt,pv,fv)	
9	Entered in cell b6: =rate(b1,b2,b3,b4)	

Applying Compounding to Things Other Than Money

While this chapter focuses on moving money through time at a given interest rate, the concept of compounding applies to almost anything that grows. For example, let's assume you're

interested in knowing how big the market for wireless printers will be in 5 years and assume the demand for them is expected to grow at a rate of 25 percent per year over the next 5 years. We can calculate the future value of the market for printers using the same formula we used to calculate future value for a sum of money. If the market is currently 25,000 printers per year, then 25,000 would be the present value, n would be 5, r would be 25%, and substituting into equation (5-1) you would be solving FV ,

$$\begin{aligned}\text{Future value} &= \text{present value} \times (1 + r)^n \\ &= 25,000(1 + 0.20)^5 = 76,293\end{aligned}\quad (5-1)$$

In effect, you can view the interest rate, r , as a compound growth rate and solve for the number of periods it would take for something to grow to a certain level—what something will grow to in the future. Or you could solve for r , that is, solve for the rate that something would have to grow at in order to reach a target level.

Present Value

Up to this point we have been moving money forward in time; that is, we know how much we have to begin with and are trying to determine how much that sum will grow in a certain number of years when compounded at a specific rate. We are now going to look at the reverse question: What is the value in today's dollars of a sum of money to be received in the future? The answer to this question will help us determine the desirability of investment projects in Chapters 10 and 11. In this case we are moving future money back to the present. We will determine the **present value** of a lump sum, which in simple terms is the *current value of a future payment*. In fact, we will be doing nothing other than inverse compounding. The differences in these techniques come about merely from the investor's point of view. In compounding, we talked about the compound interest rate and the initial investment; in determining the present value, we will talk about the discount rate and present value of future cash flows. Determining the discount rate is the subject of Chapter 9 and can be defined as the rate of return available on an investment of equal risk to what is being discounted. Other than that, the technique and the terminology remain the same, and the mathematics are simply reversed. In equation (5-1) we were attempting to determine the future value of an initial investment. We now want to determine the initial investment or present value. By dividing both sides of equation (5-1) by $(1 + r)^n$, we get

$$\text{Present value} = \text{future value at the end of year } n \times \left[\frac{1}{(1 + r)^n} \right]$$

or

$$PV = FV_n \left[\frac{1}{(1 + r)^n} \right] \quad (5-2)$$

The term in the brackets in equation (5-2) is referred to as the **present value factor**. Thus, to find the present value of a future dollar amount, all you need to do is multiply that future dollar amount times the appropriate present value factor:

$$\text{Present value} = \text{future value (present value factor)}$$

where

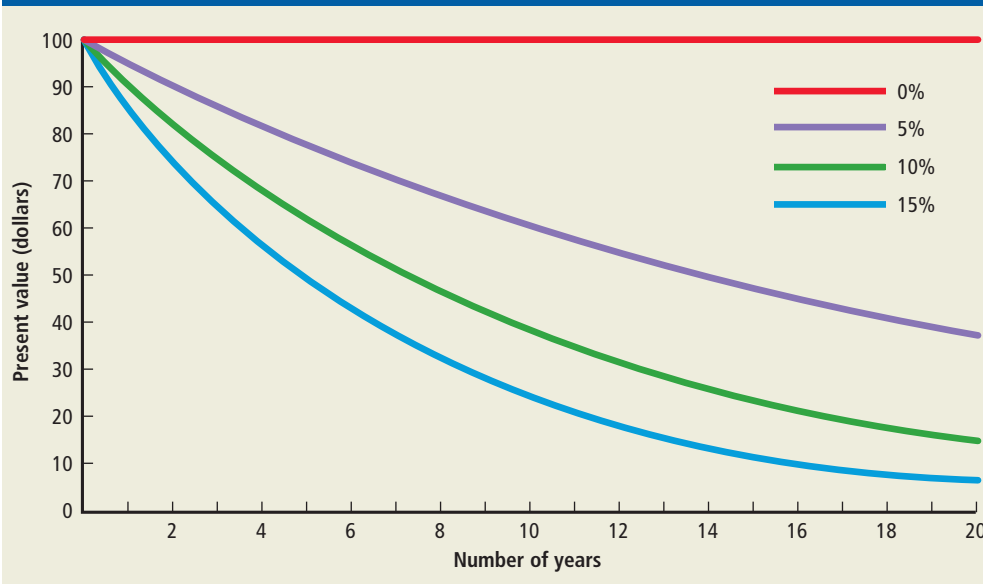
$$\text{Present value factor} = \left[\frac{1}{(1 + r)^n} \right]$$

Because the mathematical procedure for determining the present value is exactly the inverse of determining the future value, we also find that the relationships among n , r , and present value are just the opposite of those we observed in future value. The present value of a future sum of money is inversely related to both the number of years until the payment will be received and the discount rate. This relationship is shown in Figure 5-3. Although the present value equation

present value the value in today's dollars of a future payment discounted back to present at the required rate of return.

present value factor the value of $\frac{1}{(1 + r)^n}$ used as a multiplier to calculate an amount's present value.

FIGURE 5-3 The Present Value of \$100 to Be Received at a Future Date and Discounted Back to the Present at 0, 5, 10, and 15 Percent



[equation (5-2)] is used extensively to evaluate new investment proposals, it should be stressed that the equation is actually the same as the future value, or compounding equation [equation (5-1)], only it solves for present value instead of future value.

EXAMPLE 5.4

Calculating the discounted value to be received in 10 years

What is the present value of \$500 to be received 10 years from today if our discount rate is 6 percent?

STEP 1: FORMULATE A SOLUTION STRATEGY

The present value to be received can be calculated using equation (5-2) as follows:

$$\text{Present value} = FV_n \left[\frac{1}{(1 + r)^n} \right] \quad (5-2)$$

STEP 2: CRUNCH THE NUMBERS

Substituting $FV = \$500$, $n = 10$, and $r = 6$ percent into equation (5-2), we find:

$$\begin{aligned} \text{Present value} &= \$500 \left[\frac{1}{(1 + 0.06)^{10}} \right] \\ &= \$500(0.5584) \\ &= \$279.20 \end{aligned}$$

STEP 3: ANALYZE YOUR RESULTS

Thus, the present value of the \$500 to be received in 10 years is \$279.20.

CALCULATOR SOLUTION

Data Input	Function Key
10	N
6	I/Y
-500	FV
0	PMT
Function Key	Answer
CPT	
PV	279.20

EXAMPLE 5.5

Calculating the present value of a savings bond

You're on vacation in a rather remote part of Florida and see an advertisement stating that if you take a sales tour of some condominiums "you will be given \$100 just for taking the tour." However, the \$100 that you get is in the form of a savings bond that

will not pay you the \$100 for 10 years. What is the present value of \$100 to be received 10 years from today if your discount rate is 6 percent?

STEP 1: FORMULATE A SOLUTION STRATEGY

The present value of our savings bond can be computed using equation (5-2) as follows:

$$\text{Present value} = FV_n \left[\frac{1}{(1 + r)^n} \right] \quad (5-2)$$

STEP 2: CRUNCH THE NUMBERS

Substituting $FV = \$100$, $n = 10$, and $r = 6$ percent into equation (5-2), we compute the present value as follows:

$$\begin{aligned} \text{Present value} &= \$100 \left[\frac{1}{(1 + 0.06)^{10}} \right] \\ &= \$100(0.5584) \\ &= \$55.84 \end{aligned}$$

STEP 3: ANALYZE YOUR RESULTS

Thus, the value in today's dollars of that \$100 savings bond is only \$55.84.

CALCULATOR SOLUTION

Data Input	Function Key
10	N
6	I/Y
-100	FV
0	PMT
Function Key	Answer
CPT	
PV	55.84

CAUTIONARY TALE

FORGETTING PRINCIPLE 4: MARKET PRICES ARE GENERALLY RIGHT

In the Cautionary Tale for Chapter 2, we looked at the role the mortgage crisis played in the recent financial collapse from the viewpoint of conflicts of interest and failed corporate governance. But there are many lenses through which we can look to analyze the crisis. One such lens is the principle of efficient markets.

In 2007, several U.S. real estate markets entered a housing bubble. To look more closely at the underlying factors that led to the recent housing bubble (and burst), let's take a step back for a moment.

Beginning in the mid-1990s, the federal government made moves to relax conventional lending standards. In one such move, the government required the Federal National Mortgage Association, commonly known as Fannie Mae, and the Federal Home Loan Mortgage Corporation, known as Freddie Mac, to increase their holdings of loans to low- and moderate-income borrowers. Then in 1999, the U.S. Department of Housing and Urban Development (HUD) regulations required Fannie Mae and Freddie Mac to accept more loans with little or no down payment. As a result, the government had opened the door to very risky loans that would not have been made without this government action.

After the 2001 terrorist attack on the World Trade Center, the government made another move that acted against what we know about competitive markets. The Fed lowered short-term interest rates to ensure that the economy did not seize up. These low, short-term interest rates made adjustable rate loans

with low down payments highly attractive to homebuyers. As a result of the low interest rate, when individuals took out variable rate mortgages, they often qualified for bigger mortgages than they could have afforded during a normal interest rate period. But in 2005 and 2006, to control inflation, the Fed returned these short-term interest rates to higher levels and the adjustable rates reset, causing the monthly payments on these loans to increase. Housing prices began to fall and defaults soared.

These actions prevented supply and demand from acting naturally. As a result, housing prices were unnaturally inflated and the listed value of the mortgages, when packaged as securities, was a poor indicator of their actual worth. When homeowners defaulted on their loans in spades, investors were left holding the bad mortgages. These defaulted mortgages also led to a lot more houses on the market that the banks couldn't sell, which led to the market drying up, as it then became very difficult for anyone to get a new loan.

We now know that these events put into motion the housing bubble that contributed to our recent economic downturn. Competitive markets operate with natural forces of supply and demand, and while they tend to eliminate huge returns, competitive markets can also help to prevent the occurrence of short-lived false values—such as the temporarily low monthly interest payments for new homebuyers—that lead to an eventual crash. If we take one lesson away from this, it should be this: Don't mess with efficient markets. If the markets move interest rates to higher levels, it's for a reason.

Again, we have only one present-value–future-value equation; that is, equations (5-1) and (5-2) are different formats of the same equation. We have introduced them as separate equations to simplify our calculations; in one case we are determining the value in future dollars, and in the other case the value in today's dollars. In either case, the reason is the same: to compare values on alternative investments and to recognize that the value of a dollar received today is not the same as that of a dollar received at some future date. In other words, we must measure the dollar values in dollars of the same time period. For example, if we looked at these projects—one that promised \$1,000 in 1 year, one that promised \$1,500 in 5 years, and one that promised \$2,500 in 10 years—the concept of present value allows us to bring their flows back to the present and make those projects comparable. Moreover, because all present values are comparable (they are all measured in dollars of the same time period), we can add and subtract the present value of inflows and outflows to determine the present value of an investment. Let's now look at an example of an investment that has two cash flows in different time periods and determine the present value of this investment.

STEP 1**CALCULATOR SOLUTION**

Data Input	Function Key
7	N
6	I/Y
−1,000	FV
0	PMT
Function Key	Answer
CPT	
PV	665.06

STEP 2**CALCULATOR SOLUTION**

Data Input	Function Key
10	N
6	I/Y
−1,000	FV
0	PMT
Function Key	Answer
CPT	
PV	558.39

STEP 3

Add the two present values that you just calculated together:

$$\begin{array}{r}
 \$ 665.06 \\
 \quad 558.39 \\
 \hline
 \$1,223.45
 \end{array}$$

EXAMPLE 5.6**Calculating the present value of an investment**

What is the present value of an investment that yields \$1,000 to be received in 7 years and \$1,000 to be received in 10 years if the discount rate is 6 percent?

STEP 1: FORMULATE A SOLUTION STRATEGY

The present value of our investment can be computed using equation (5-2) for each yield, then adding these values together as follows:

$$\text{Present value} = FV_n \left[\frac{1}{(1 + r)^n} \right] + FV_n \left[\frac{1}{(1 + r)^n} \right] \quad (5-2)$$

STEP 2: CRUNCH THE NUMBERS

Substituting into equation (5-2), we compute the future value as follows:

$$\begin{aligned}
 \text{Present value} &= \$1,000 \left[\frac{1}{(1 + 0.06)^7} \right] + \$1,000 \left[\frac{1}{(1 + 0.06)^{10}} \right] \\
 &= \$665.06 + \$558.39 = \$1,223.45
 \end{aligned}$$

STEP 3: ANALYZE YOUR RESULTS

Again, present values are comparable because they are measured in the same time period's dollars.

With a financial calculator, this becomes a three-step solution, as shown in the margin. First, you'll solve for the present value of the \$1,000 received at the end of 7 years, then you'll solve for the present value of the \$1,000 received at the end of 10 years, and finally, you'll add the two present values together. Remember, once you've found the present value of those future cash flows you can add them together because they're measured in the same period's dollars.

CAN YOU DO IT?**SOLVING FOR THE PRESENT VALUE WITH TWO FLOWS IN DIFFERENT YEARS**

What is the present value of an investment that yields \$500 to be received in 5 years and \$1,000 to be received in 10 years if the discount rate is 4 percent?

(The solution can be found on page 158.)

Concept Check

1. Principle 2 states that “money has a time value.” Explain this statement.
2. How does compound interest differ from simple interest?
3. Explain the formula $FV_n = PV(1 + r)^n$.
4. Why is the present value of a future sum always less than that sum’s future value?

Annuities

An **annuity** is a *series of equal dollar payments for a specified number of years*. When we talk about annuities, we are referring to **ordinary annuities** unless otherwise noted. With an ordinary annuity *the payments occur at the end of each period*. Because annuities occur frequently in finance—for example, as bond interest payments—we treat them specially. Although compounding and determining the present value of an annuity can be dealt with using the methods we have just described, these processes can be time-consuming, especially for larger annuities. Thus, we have modified the single cash flow formulas to deal directly with annuities.

b² Understand annuities.

annuity a series of equal dollar payments made for a specified number of years.

ordinary annuity an annuity where the cash flows occur at the end of each period.

Compound Annuities

A **compound annuity** involves *depositing or investing an equal sum of money at the end of each year for a certain number of years and allowing it to grow*. Perhaps we are saving money for education, a new car, or a vacation home. In any case, we want to know how much our savings will have grown at some point in the future.

Actually, we can find the answer by using equation (5-1), our compounding equation, and compounding each of the individual deposits to its future value. For example, if to provide for a college education we are going to deposit \$500 at the end of each year for the next 5 years in a bank where it will earn 6 percent interest, how much will we have at the end of 5 years? Compounding each of these values using equation (5-1), we find that we will have \$2,818.50 at the end of 5 years.

$$\begin{aligned}
 FV_5 &= \$500(1 + 0.06)^4 + \$500(1 + 0.06)^3 + \$500(1 + 0.06)^2 + \$500(1 + 0.06) + \$500 \\
 &= \$500(1.262) + \$500(1.191) + \$500(1.124) + \$500(1.060) + \$500 \\
 &= \$631.00 + \$595.50 + \$562.00 + \$530.00 + \$500.00 \\
 &= \$2,818.50
 \end{aligned}$$

By examining the mathematics involved and the graph of the movement of money through time in Table 5-1, we can see that all we are really doing is adding up the future values of different cash flows that initially occurred in different time periods. Fortunately, there is also an equation that helps us calculate the future value of an annuity:

$$\begin{aligned}
 \text{Future value of an annuity} &= PMT \left[\frac{\text{future value factor} - 1}{r} \right] \\
 &= PMT \left[\frac{(1 + r)^n - 1}{r} \right]
 \end{aligned}
 \tag{5-3}$$

TABLE 5-1 Growth of a 5-Year, \$500 Annuity Compounded at 6 Percent						
	$r = 6\%$					
YEAR	0	1	2	3	4	5
Dollar deposits at end of year		500	500	500	500	500
						\$ 500.00
						530.00
						562.00
						595.50
						631.00
Future value of the annuity						<u>\$2,818.50</u>

DID YOU GET IT?

SOLVING FOR THE PRESENT VALUE WITH TWO FLOWS IN DIFFERENT YEARS

There are several different ways you can solve this problem—using the mathematical formulas, a financial calculator, or a spreadsheet—each one giving you the same answer.

- 1. Using the Mathematical Formula.** Substituting the values of $n = 5$, $r = 4$ percent, and $FV_5 = \$500$; and $n = 10$, $r = 4$ percent, and $FV_{10} = \$1,000$ into equation (5-2) and adding these values together, we find

$$\begin{aligned}\text{Present value} &= \$500 \left[\frac{1}{(1 + 0.04)^5} \right] + \$1,000 \left[\frac{1}{(1 + 0.04)^{10}} \right] \\ &= \$500(0.822) + \$1,000(0.676) \\ &= \$411 + \$676 = \$1,087\end{aligned}$$

- 2. Using a Financial Calculator.** Again, it is a three-step process. First calculate the present value of each cash flow individually, and then add the present values together.

STEP 1

CALCULATOR SOLUTION

Data Input	Function Key
5	N
4	I/Y
-500	FV
0	PMT
Function Key	Answer
CPT	
PV	410.96

STEP 2

CALCULATOR SOLUTION

Data Input	Function Key
10	N
4	I/Y
-1,000	PV
0	PMT
Function Key	Answer
CPT	
PV	675.56

STEP 3

Add the two present values that you just calculated together:

$$\begin{array}{r} \$ 410.96 \\ 675.56 \\ \hline \$1,086.52 \end{array}$$

- 3. Using an Excel Spreadsheet.** Using Excel, the cash flows are brought back to the present using the `=PV` function. If the future values were entered as positive values, our answer will come out as a negative number.

annuity future value factor the value of $\left[\frac{(1 + r)^n - 1}{r} \right]$ used as a multiplier to calculate the future value of an annuity.

To simplify our discussion, we will refer to the value in brackets in equation (5-3) as the **annuity future value factor**. It is defined as $\left[\frac{(1 + r)^n - 1}{r} \right]$.

Using this new notation, we can rewrite equation (5-3) as follows:

$$FV_n = PMT \left[\frac{(1 + r)^n - 1}{r} \right] = PMT(\text{annuity future value factor})$$

Rather than asking how much we will accumulate if we deposit an equal sum in a savings account each year, a more common question to ask is how much we must deposit each year to accumulate a certain amount of savings. This problem frequently occurs with respect to saving for large expenditures.

For example, if we know that we need \$10,000 for college in 8 years, how much must we deposit in the bank at the end of each year at 6 percent interest to have the college money ready? In this case we know the values of n , r , and FV_n in equation (5-3); what we do not know is the value of PMT . Substituting these example values in equation (5-3), we find

$$\$10,000 = PMT \left[\frac{(1 + 0.06)^8 - 1}{0.06} \right]$$

$$\$10,000 = PMT(9.8975)$$

$$\frac{\$10,000}{9.8975} = PMT$$

$$\$1,010.36 = PMT$$

CALCULATOR SOLUTION

Data Input	Function Key
8	N
6	I/Y
-10,000	FV
0	PV
Function Key	Answer
CPT	
PMT	1,010.36

Thus, we must deposit \$1,010.36 in the bank at the end of each year for 8 years at 6 percent interest to accumulate \$10,000 at the end of 8 years.

EXAMPLE 5.7**Calculating deposit amount to accumulate \$5,000**

How much must we deposit in an 8 percent savings account at the end of each year to accumulate \$5,000 at the end of 10 years?

STEP 1: FORMULATE A SOLUTION STRATEGY

In order to determine the amount we must deposit, we have to use equation (5-3) as follows:

$$FV = PMT \left[\frac{(1 + r)^n - 1}{r} \right] \quad (5-3)$$

STEP 2: CRUNCH THE NUMBERS

Substituting into equation (5-3), we compute the future value as follows:

$$\$5,000 = PMT \left[\frac{(1 + 0.06)^8 - 1}{0.06} \right]$$

$$\$5,000 = PMT(14.4866)$$

$$\frac{5,000}{14.4866} = PMT = \$345.15$$

STEP 3: ANALYZE YOUR RESULTS

Thus, we must deposit \$345.15 per year for 10 years at 8 percent to accumulate \$5,000.

CALCULATOR SOLUTION

Data Input	Function Key
10	N
8	I/Y
-5,000	FV
0	PV
Function Key	Answer
CPT	
PMT	345.15

The Present Value of an Annuity

Pension payments, insurance obligations, and the interest owed on bonds all involve annuities. To compare these three types of investments we need to know the present value of each. For example, if we wish to know what \$500 received at the end of each of the next 5 years is worth today given a discount rate of 6 percent, we can simply substitute the appropriate values into equation (5-2), such that

$$\begin{aligned}
 PV &= \$500 \left[\frac{1}{(1 + 0.06)^1} \right] + \$500 \left[\frac{1}{(1 + 0.06)^2} \right] + \$500 \left[\frac{1}{(1 + 0.06)^3} \right] \\
 &\quad + \$500 \left[\frac{1}{(1 + 0.06)^4} \right] + \$500 \left[\frac{1}{(1 + 0.06)^5} \right] \\
 &= \$500(0.943) + \$500(0.890) + \$500(0.840) + \$500(0.792) + \$500(0.747) \\
 &= \$2,106
 \end{aligned}$$

Thus, the present value of this annuity is \$2,106.00. By examining the mathematics involved and the graph of the movement of money through time in Table 5-2, we can see that all we are really doing is adding up the present values of different cash flows that initially occurred in different time periods. Fortunately, there is also an equation that helps us calculate the present value of an annuity:

$$\begin{aligned}
 \text{Present value of an annuity} &= PMT \left[\frac{1 - \text{present value factor}}{r} \right] \\
 &= PMT \left[\frac{1 - (1 + r)^{-n}}{r} \right] \quad (5-4)
 \end{aligned}$$

TABLE 5-2 Illustration of a 5-Year, \$500 Annuity Discounted to the Present at 6 Percent

YEAR	0	1	2	3	4	5
Dollars received at end of year		500	500	500	500	500
	\$ 471.50	←				
	445.00	←				
	420.00	←				
	396.00	←				
	373.50	←				
Present value of the annuity	\$2,106.00					

annuity present value factor the value of $\left[\frac{1 - (1 + r)^{-n}}{r} \right]$ used as a multiplier to calculate the present value of an annuity

To simplify our discussion, we will refer to the value in the brackets in equation (5-4) as the **annuity present value factor**. It is defined as $\left[\frac{1 - (1 + r)^{-n}}{r} \right]$.

CALCULATOR SOLUTION

Data Input	Function Key
10	N
5	I/Y
-1,000	PMT
0	PV
Function Key	Answer
CPT	
PV	7,722

EXAMPLE 5.8**Calculating the present value of an annuity**

What is the present value of a 10-year \$1,000 annuity discounted back to the present at 5 percent?

STEP 1: FORMULATE A SOLUTION STRATEGY

The present value of the annuity can be computed using equation (5-4) as follows:

$$PV = PMT \left[\frac{1 - (1 + r)^{-n}}{r} \right] \quad (5-4)$$

STEP 2: CRUNCH THE NUMBERS

Substituting into equation (5-4), we compute the present value as follows:

$$\begin{aligned} PV &= \$1,000 \left[\frac{1 - (1 + 0.05)^{-10}}{0.05} \right] \\ &= \$1,000(7.722) \\ &= \$7,722 \end{aligned}$$

STEP 3: ANALYZE YOUR RESULTS

Thus, the present value of this annuity is \$7,722.

CALCULATOR SOLUTION

Data Input	Function Key
5	N
8	I/Y
-5,000	PV
0	FV
Function Key	Answer
CPT	
PMT	1,252

When we solve for PMT , the financial interpretation of this action would be: How much can be withdrawn, perhaps as a pension or to make loan payments, from an account that earns r percent compounded annually for each of the next n years if we wish to have nothing left at the end of n years? For example, if we have \$5,000 in an account earning 8 percent interest, how large of an annuity can we draw out each year if we want nothing left at the end of 5 years? In this case the present value, PV , of the annuity is \$5,000, $n = 5$ years, $r = 8$ percent, and PMT is unknown. Substituting this into equation (5-4), we find

$$\begin{aligned} \$5,000 &= PMT \left[\frac{1 - (1 + 0.08)^{-5}}{0.08} \right] = PMT(3.993) \\ \$1,252 &= PMT \end{aligned}$$

Thus, this account will fall to zero at the end of 5 years if we withdraw \$1,252 at the end of each year.

Annuities Due

Annuities due are really just *ordinary annuities in which all the annuity payments have been shifted forward by 1 year*. Compounding them and determining their future and present value is actually quite simple. With an annuity due, each annuity payment occurs at the beginning of each period rather than at the end of the period. Let's first look at how this affects our compounding calculations.

Because an annuity due merely shifts the payments from the end of the year to the beginning of the year, we now compound the cash flows for one additional year. Therefore, the compound sum of an annuity due is simply

$$\begin{aligned}\text{Future value of an annuity due} &= \text{future value of an annuity} \times (1 + r) \\ FV_n(\text{annuity due}) &= PMT \left[\frac{(1 + r)^n - 1}{r} \right] (1 + r) \quad (5-5)\end{aligned}$$

In an earlier example on saving for college, we calculated the value of a 5-year ordinary annuity of \$500 invested in the bank at 6 percent to be \$2,818.50. If we now assume this to be a 5-year annuity due, its future value increases from \$2,818.50 to \$2,987.66.

$$\begin{aligned}FV_5 &= \$500 \left[\frac{(1 + 0.06)^5 - 1}{0.06} \right] (1 + 0.06) \\ &= \$500(5.637093)(1.06) \\ &= \$2,987.66\end{aligned}$$

Likewise, with the present value of an annuity due, we simply receive each cash flow 1 year earlier—that is, we receive it at the beginning of each year rather than at the end of each year. Thus, because each cash flow is received 1 year earlier, it is discounted back for one less period. To determine the present value of an annuity due, we merely need to find the present value of an ordinary annuity and multiply that by $(1 + r)$, which in effect cancels out 1 year's discounting.

$$\begin{aligned}\text{Present value of an annuity due} &= \text{present value of an annuity} \times (1 + r) \\ PV(\text{annuity due}) &= PMT \left[\frac{1 - (1 + r)^{-n}}{r} \right] (1 + r) \quad (5-6)\end{aligned}$$

Reexamining the earlier college saving example in which we calculated the present value of a 5-year ordinary annuity of \$500 given a discount rate of 6 percent, we now find that if it is an annuity due rather than an ordinary annuity, the present value increases from \$2,106 to \$2,232.55.

$$\begin{aligned}PV &= \$500 \left[\frac{1 - (1 + 0.06)^{-5}}{0.06} \right] (1 + 0.06) \\ &= \$500(4.21236)(1.06) \\ &= \$2,232.55\end{aligned}$$

With a financial calculator, first treat it as if it were an ordinary annuity and find the future or present value. Then multiply the present value times $(1 + r)$, in this case times 1.06; this is shown in the margin.

The result of all this is that both the future and present values of an annuity due are larger than those of an ordinary annuity because in each case all payments are received earlier. Thus, when *compounding*, an annuity due compounds for 1 additional year, whereas when *discounting*, an annuity due discounts for 1 less year. Although annuities due are used with some frequency in accounting, their usage is quite limited in finance. Therefore, in the remainder of this text, whenever the term *annuity* is used, you should assume that we are referring to an ordinary annuity.

annuity due an annuity in which the payments occur at the beginning of each period.

Future value of an annuity due:

STEP 1

CALCULATOR SOLUTION

Data Input	Function Key
5	N
6	I/Y
0	PV
−500	PMT
Function Key	Answer
CPT	
FV	2,818.55

STEP 2

$$\begin{aligned}&\$2,818.55 \\ &\times 1.06 \\ &\hline &\$2,987.66\end{aligned}$$

Present value of an annuity due:

STEP 1

CALCULATOR SOLUTION

Data Input	Function Key
5	N
6	I/Y
−500	PMT
0	FV
Function Key	Answer
CPT	
PV	2,106.18

STEP 2

$$\begin{aligned}&\$2,106.18 \\ &\times 1.06 \\ &\hline &\$2,232.55\end{aligned}$$

EXAMPLE 5.9**Calculating the Lotto payments on \$2 million**

Forty-three states run lotteries and most of those lotteries are similar: You must select 6 out of 44 numbers correctly in order to win the jackpot. If you come close, there are some significantly lesser prizes, which we ignore for now. For each million dollars in the lottery jackpot, you receive \$50,000 per year for 20 years, and your chance of winning is 1 in 7.1 million. A recent advertisement for a particular state lottery went as follows: “Okay, you got two kinds of people. You’ve got the kind who play Lotto all the time, and the kind who play Lotto some of the time. You know, like only on a Saturday when they stop in at the store on the corner for some peanut butter cups and diet soda and the jackpot happens to be really big. I mean, my friend Ned? He’s like ‘Hey, it’s only \$2 million this week.’ Well, hellloooo, anybody home? I mean, I don’t know about you, but I wouldn’t mind having a measly 2 mill coming *my* way. . . .”

What is the present value of these payments?

STEP 1: FORMULATE A SOLUTION STRATEGY

The answer to this question depends on what assumption you make about the time value of money. In this case, let’s assume that your required rate of return on an investment with this level of risk is 10 percent. Keeping in mind that the Lotto is an annuity due—that is, on a \$2 million lottery you would get \$100,000 immediately and \$100,000 at the end of each of the next 19 years. Thus, the present value of this 20-year annuity due discounted back to present at 10 percent becomes:

$$PV(\text{annuity due}) = PMT \left[\frac{1 - (1 + r)^{-n}}{r} \right]$$

STEP 2: CRUNCH THE NUMBERS

Solving this is a two-step process. In step 1, you treat the jackpot as if it were an ordinary annuity and find the present value. Then multiply the present value times $(1 + r)$, in this case times 1.10.

Substituting into equation (5-6), we compute the present value as follows:

$$\begin{aligned} PV &= \$100,000 \left[\frac{1 - (1 + 0.10)^{-20}}{0.10} \right] (1 + 0.10) \\ &= \$100,000(8.51356)(1.10) \\ &= \$936,492 \end{aligned}$$

STEP 3: ANALYZE YOUR RESULTS

Thus, the present value of the \$2 million Lotto jackpot is less than \$1 million if 10 percent is the discount rate. Moreover, because the chance of winning is only 1 in 7.1 million, the expected value of each dollar “invested” in the lottery is only $(1/7.1 \text{ million}) \times (\$936,492) = 13.19\text{¢}$. That is, for every dollar you spend on the lottery you should expect to get, on average, about 13 cents back—not a particularly good deal. Although this ignores the minor payments for coming close, it also ignores taxes. In this case, it looks like “my friend Ned” is doing the right thing by staying clear of the lottery. Obviously, the main value of the lottery is entertainment. Unfortunately, without an understanding of the time value of money, it can sound like a good investment.

Present value of an annuity due:

STEP 1**CALCULATOR SOLUTION**

Data Input	Function Key
20	N
10	I/Y
−100,000	PMT
0	FV
Function Key	Answer
CPT	
PV	851,356

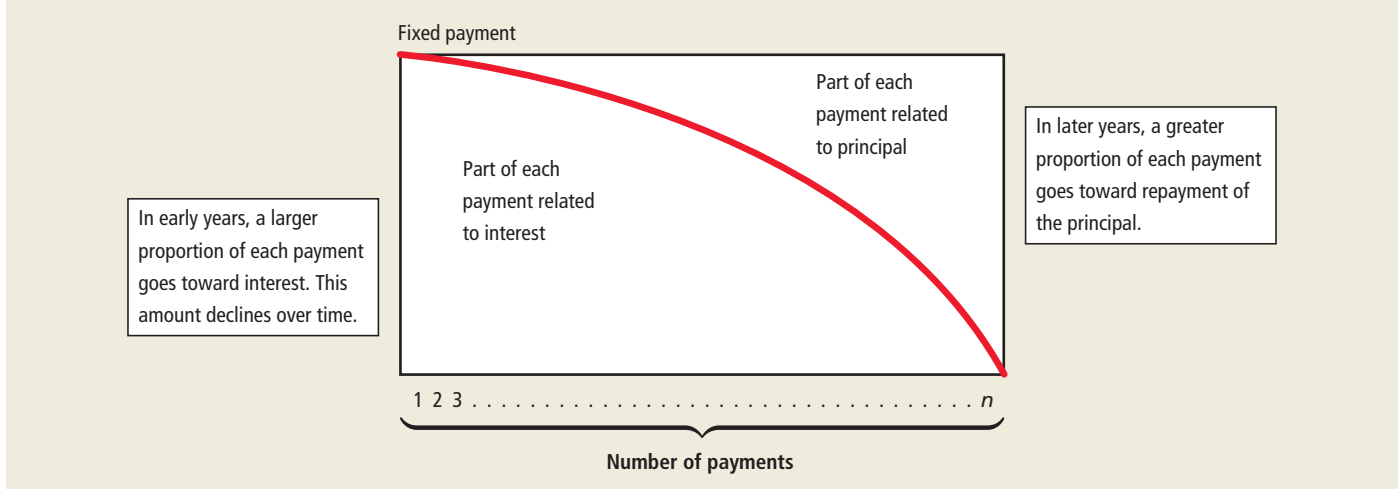
STEP 2

$$\begin{array}{r} \$851,356 \\ \times 1.10 \\ \hline \$936,492 \end{array}$$

Amortized Loans

The procedure of solving for PMT , the annuity payment value when r , n , and PV are known, is also used to determine what payments are associated with paying off a loan in equal installments over time. *Loans that are paid off this way, in equal periodic payments,* are called **amortized loans**. Actually, the word *amortization* comes from the Latin word meaning “about to die.” When you pay off a loan using regular, fixed payments, that loan is amortized. Although the payments are fixed, different amounts of each payment are applied

amortized loan a loan that is paid off in equal periodic payments.

FIGURE 5-4 The Amortization Process

toward the principal and the interest. With each payment, you owe a bit less toward the principal. As a result, the amount that has to go toward the interest payment declines with each payment, whereas the portion of each payment that goes toward the principal increases. Figure 5-4 illustrates the process of amortization.

For example, suppose a firm wants to purchase a piece of machinery. To do this, it borrows \$6,000 to be repaid in four equal payments at the end of each of the next 4 years, and the interest rate that is paid to the lender is 15 percent on the outstanding portion of the loan. To determine what the annual payments associated with the repayment of this debt will be, we simply use equation (5-4) and solve for the value of PMT , the annual annuity. In this case, we know PV , r , and n . PV , the present value of the annuity, is \$6,000; r , the annual interest rate, is 15 percent; and n , the number of years for which the annuity will last, is 4 years. PMT , the annuity payment received (by the lender and paid by the firm) at the end of each year, is unknown. Substituting these values into equation (5-4), we find

$$\$6,000 = PMT \left[\frac{1 - (1 + 0.15)^{-4}}{0.15} \right]$$

$$\$6,000 = PMT(2.85498)$$

$$\$2,101.59 = PMT$$

To repay the principal and interest on the outstanding loan in 4 years, the annual payments would be \$2,101.59. The breakdown of interest and principal payments is given in the *loan amortization schedule* in Table 5-3, with very minor rounding error. As you can see, the interest portion of the payment declines each year as the loan outstanding balance declines.

CALCULATOR SOLUTION

Data Input	Function Key
4	N
15	I/Y
-6,000	PV
0	FV
Function Key	Answer
CPT	
PMT	2,101.59

TABLE 5-3 Loan Amortization Schedule Involving a \$6,000 Loan at 15 Percent to Be Repaid in 4 Years

Year	Annuity	Interest Portion of the Annuity ^a	Repayment of the principal Portion of the Annuity ^b	Outstanding Loan Balance After the Annuity Payment
0	0	0	0	\$6,000.00
1	\$2,101.59	\$900.00	\$1,201.59	4,798.41
2	2,101.59	719.76	1,381.83	3,416.58
3	2,101.59	512.49	1,589.10	1,827.48
4	2,101.59	274.12	1,827.48	

^aThe interest portion of the annuity is calculated by multiplying the outstanding loan balance at the beginning of the year by the interest rate of 15 percent. Thus, for year 1 it was $\$6,000 \times 0.15 = \900.00 , for year 2 it was $\$4,798.41 \times 0.15 = \719.76 , and so on.

^bRepayment of the principal portion of the annuity was calculated by subtracting the interest portion of the annuity (column 2) from the annuity (column 1).

Now let's use a spreadsheet to look at a loan amortization problem and calculate the monthly mortgage payment, and then determine how much of a specific payment goes toward interest and principal.

To buy a new house you take out a 25-year mortgage for \$100,000. What will your monthly payments be if the interest rate on your mortgage is 8 percent? To solve this problem, you must first convert the annual rate of 8 percent into a monthly rate by dividing it by 12. Second, you have to convert the number of periods into months by multiplying 25 years times 12 months per year for a total of 300 months. We'll look at this process in more detail in the next section.

	A	B
1	interest rate (rate) =	0.6667%
2	number of periods (nper) =	300
3	present value (pv) =	\$100,000
4	future value (fv) =	\$0
5		
6	payment (pmt) =	(\$771.82)
7		
8	Excel formula =pmt(rate,nper,pv,fv)	
9	Entered in cell b6: =pmt((8/12)%,b2,b3,b4)	

You can also use Excel to calculate the interest and principal portion of any loan amortization payment. You can do this using the following Excel functions:

CALCULATION	FUNCTION
Interest portion of payment	= IPMT(rate, per, nper, pv, fv)
Principal portion of payment	= PPMT(rate, per, nper, pv, fv)
where per refers to the number of an individual periodic payment	

For this example, you can determine how much of the 48th monthly payment goes toward interest and principal as follows,

	A	B
1	interest rate (rate) =	0.6667%
2	payment number (per) =	48
3	number of periods (nper) =	300
4	present value (pv) =	\$100,000
5	future value (fv) =	\$0
6		
7	Interest portion of the 48th payment =	(\$628.12)
8	Principal portion of the 48th loan payment =	(\$143.69)
9		
10	Entered values in cell b7: =IPMT((8/12)%,b2,b3,b4,b5)	
11	Entered values in cell b8: =PPMT((8/12)%,b2,b3,b4,b5)	

Concept Check

1. What is an amortized loan?
2. What functions in Excel help determine the amount of interest and principal in a mortgage payment?

Making Interest Rates Comparable

In order to make intelligent decisions about where to invest or borrow money, it is important that we make the stated interest rates comparable. Unfortunately, some rates are quoted as compounded annually, whereas others are quoted as compounded quarterly or compounded daily. But it is not fair to compare interest rates with different compounding periods to each other. Thus, the only way interest rates can logically be compared is to convert them to a common compounding period.

In order to understand the process of making different interest rates comparable, it is first necessary to define the nominal, or quoted, interest rate. The nominal, or quoted, rate is the rate of interest stated on the contract. For example, if you shop around for loans and are quoted 8 percent compounded annually and 7.85 percent compounded quarterly, then 8 percent and 7.85 percent would both be nominal rates. Unfortunately, because on one rate the interest is compounded annually, but on the other interest is compounded quarterly, the two rates are not comparable. In fact, it is never appropriate to compare nominal rates *unless* they include the same number of compounding periods per year. To make them comparable, we must calculate their equivalent rate at some common compounding period. We do this by calculating the **effective annual rate (EAR)**. This is *the annual compound rate that produces the same return as the nominal or quoted rate*.

Let's assume that you are considering borrowing money from a bank at 12 percent compounded monthly. If you borrow \$1 at 1 percent per month for 12 months you'd owe,

$$\$1.00(1.01)^{12} = \$1.126825$$

In effect, you are borrowing at 12.6825 percent rather than just by 12 percent. Thus, the EAR for 12 percent compounded monthly is 12.6825. It tells us the annual rate that would produce the same loan payments as the nominal rate. In other words, you'll end up with the same monthly payments if you borrow at 12.6825 percent compounded annually or 12 percent compounded monthly. Thus, 12.6825 is the effective annual rate (EAR) for 12 percent compounded monthly.

Generalizing on this process, we can calculate the EAR using the following equation:

$$EAR = \left(1 + \frac{\text{quoted rate}}{m} \right)^m - 1 \quad (5-7)$$

where *EAR* is the effective annual rate and *m* is the number of compounding periods within a year. Given the wide variety of compounding periods used by businesses and banks, it is important to know how to make these rates comparable so that logical decisions can be made.

EXAMPLE 5.10 Calculating the EAR on a credit card

You've just received your first credit card and the problem is the rate. It looks pretty high to you. The quoted rate is 21.7 percent, and when you look closer, you notice that the interest is compounded daily. What's the EAR, or effective annual rate, on your credit card?

STEP 1: FORMULATE A SOLUTION STRATEGY

To calculate the EAR we can use equation (5-7) as follows:

$$EAR = \left[1 + \frac{\text{quoted rate}}{m} \right]^m - 1$$



3 Determine the future or present value of a sum when there are nonannual compounding periods.

effective annual rate (EAR) the annual compound rate that produces the same return as the nominal, or quoted, rate when something is compounded on a nonannual basis. In effect, the EAR provides the true rate of return.

STEP 2: CRUNCH THE NUMBERS

Substituting into equation (5-7), we compute the EAR as follows:

$$EAR = \left[1 + \frac{0.217}{365} \right]^{365} - 1$$

$$EAR = 1.242264 - 1 = 0.242264, \text{ or } 24.2264 \text{ percent.}$$

STEP 3: ANALYZE YOUR RESULTS

Thus, in reality, the effective annual rate is actually 24.2264 percent.

Finding Present and Future Values with Nonannual Periods

The same logic that applies to calculating the EAR also applies to calculating present and future values when the periods are semiannual, quarterly, or any other nonannual period. Previously when we moved money through time we assumed that the cash flows occurred annually and the compounding or discounting period was always annual. However, it need not be, as evidenced by the fact that bonds generally pay interest semiannually and most mortgages require monthly payments.

For example, if we invest our money for 5 years at 8 percent interest compounded semiannually, we are really investing our money for ten 6-month periods during which we receive 4 percent interest each period. If it is 8 percent compounded quarterly for 5 years, we receive 2 percent interest per period for twenty 3-month periods. This process can easily be generalized, giving us the following formula for finding the future value of an investment for which interest is compounded in nonannual periods:

$$FV_n = PV \left[1 + \frac{r}{m} \right]^{m \cdot n}$$

where FV_n = the future value of the investment at the end of n years

n = the number of years during which the compounding occurs

r = annual interest (or discount) rate

PV = the present value or original amount invested at the beginning of the first year

m = the number of times compounding occurs during the year

In fact, all the formulas for moving money through time can be easily modified to accommodate nonannual periods. In each case, we begin with the formulas we introduced in this chapter, and make two adjustments—the first where n , the number of years, appears, and the second where r , the annual interest rate, appears. Thus, the adjustment involves two steps:

- ◆ n becomes the number of periods or n (the number of years) times m (the number of times compounding occurs per year). Thus, if it is monthly compounding for 10 years, n becomes $10 \times 12 = 120$ months in 10 years, and if it is daily compounding over 10 years, it becomes $10 \times 365 = 3,650$ days in 10 years.

CAN YOU DO IT?

HOW MUCH CAN YOU AFFORD TO SPEND ON A HOUSE? AN AMORTIZED LOAN WITH MONTHLY PAYMENTS

You've been house shopping and aren't sure how big a house you can afford. You figure you can handle monthly mortgage payments of \$1,250 and you can get a 30-year loan at 6.5 percent. How big of a mortgage can you afford? In this problem, you are solving for PV , which is the amount of money you can borrow today.

(The solution can be found on page 168.)

TABLE 5-4 The Value of \$100 Compounded at Various Intervals

FOR 1 YEAR AT r PERCENT				
$r =$	2%	5%	10%	15%
Compounded annually	\$102.00	\$105.00	\$110.00	\$115.00
Compounded semiannually	102.01	105.06	110.25	115.56
Compounded quarterly	102.02	105.09	110.38	115.87
Compounded monthly	102.02	105.12	110.47	116.08
Compounded weekly (52)	102.02	105.12	110.51	116.16
Compounded daily (365)	102.02	105.13	110.52	116.18
FOR 10 YEARS AT r PERCENT				
$r =$	2%	5%	10%	15%
Compounded annually	\$121.90	\$162.89	\$259.37	\$404.56
Compounded semiannually	122.02	163.86	265.33	424.79
Compounded quarterly	122.08	164.36	268.51	436.04
Compounded monthly	122.10	164.70	270.70	444.02
Compounded weekly (52)	122.14	164.83	271.57	447.20
Compounded daily (365)	122.14	164.87	271.79	448.03

◆ r becomes the interest rate per period or r (the annual interest rate) divided by m (the number of times compounding occurs per year). Thus, if it is a 6 percent annual rate with monthly compounding, r becomes $6\% \div 12 = 0.5$ percent per month, and if it is a 6 percent annual rate compounded daily, then r becomes $(6\% \div 365)$ per day.

We can see the value of intrayear compounding by examining Table 5-4. Because “interest is earned on interest” more frequently as the length of the compounding period declines, there is an inverse relationship between the length of the compounding period and the effective annual interest rate: The shorter the compounding period is, the higher the effective interest rate will be. Conversely, the longer the compounding period is, the lower the effective interest rate will be.

EXAMPLE 5.11 Calculating the growth of an investment

If we place \$100 in a savings account that yields 12 percent compounded quarterly, what will our investment grow to at the end of 5 years?

STEP 1: FORMULATE A SOLUTION STRATEGY

The future value of our savings account can be computed using equation (5-8) as follows:

$$\text{Future value} = \text{present value} \times \left(1 + \frac{r}{m}\right)^{m \cdot n}$$

STEP 2: CRUNCH THE NUMBERS

Substituting into equation (5-8), we compute the future value as follows:

$$\begin{aligned} FV &= \$100 \left(1 + \frac{0.12}{4}\right)^{4 \cdot 5} \\ &= \$100(1.8061) = \$180.61 \end{aligned}$$

STEP 3: ANALYZE YOUR RESULTS

Thus, we will have \$180.61 at the end of 5 years. In this problem, n becomes the number of quarters in 5 years while r/m becomes the interest rate per period—in effect, 3 percent per quarter for 20 quarters.

CALCULATOR SOLUTION

Data Input	Function Key
20	N
12/4	I/Y
100	PV
0	PMT
Function Key	Answer
CPT	
FV	–180.61

DID YOU GET IT?

HOW MUCH CAN YOU AFFORD TO SPEND ON A HOUSE? AN AMORTIZED LOAN WITH MONTHLY PAYMENTS

There are several different ways you can solve this problem—using the mathematical formulas, a financial calculator, or a spreadsheet—each one giving you the same answer.

1. **Using the Mathematical Formulas.** Again, you need to multiply n times m and divide r by m , where m is the number of compounding periods per year. Thus,

$$PV = \$1,250 \left[\frac{1 - \frac{1}{\left(1 + \frac{0.065}{12}\right)^{30 \cdot 12}}}{\frac{0.065}{12}} \right]$$

$$PV = \$1,250 \left[\frac{1 - \frac{1}{(1 + 0.00541667)^{360}}}{0.00541667} \right]$$

$$PV = \$1,250 \left[\frac{1 - \frac{1}{6.99179797}}{0.00541667} \right]$$

$$PV = \$1,250(158.210816)$$

$$PV = \$197,763.52$$

2. **Using a Financial Calculator.** First, you must convert everything to months. To do this you would first convert n into months by multiplying n times m (30 times 12) and enter this into N . Next you would enter the interest rate as a monthly rate by dividing r by m and entering this number into I/Y . Finally, make sure that the value entered as PMT is a monthly figure, that is, it is the monthly payment value.

CALCULATOR SOLUTION

Data Input	Function Key
360	N
6.5/12	I/Y
-1,250	PMT
0	FV
Function Key	Answer
CPT	
PV	197,763.52

3. **Using an Excel Spreadsheet.**

	A	B
1	interest rate (rate) =	0.5417%
2	number of periods (nper) =	360
3	payment (pmt) =	1,250
4	future value (fv) =	0
5		
6	present value (pv) =	(\$197,763.52)
7	Entered values in cell b6: =PV((6.5/12)%,b2,b3,b4)	
8		

CALCULATOR SOLUTION

Data Input	Function Key
13.0/12	I/Y
10,000	PV
-400	PMT
0	FV
Function Key	Answer
CPT	
N	29.3

EXAMPLE 5.12

Calculating the future value of an investment

In 2009 the average U.S. household owed about \$10,000 in credit card debt, and the average interest rate on credit card debt was 13.0 percent. On many credit cards the minimum monthly payment is 4 percent of the debt balance. If the average household paid off 4 percent of the initial amount it owed each month, that is, made payments of \$400 each month, how many months would it take to repay this credit card debt? Use a financial calculator to solve this problem.

STEP 1: FORMULATE A SOLUTION STRATEGY

The easiest approach here is to either use your financial calculator or use Excel to solve for N , the number of periods. Using Excel, you would simply use the $=NPER$ function and input values for rate, pmt, pv, and fv. Using your financial calculator, you would simply solve for N ; however, you'll need to make sure that I/Y is expressed as a monthly rate because you are solving for the number of months necessary to repay the credit card.

STEP 2: CRUNCH THE NUMBERS

The solution, using a financial calculator, appears in the margin. You'll notice in the solution that the value for PMT goes in as a negative.

STEP 3: ANALYZE YOUR RESULTS

The answer is over 29 months before you pay off your credit card if you pay off 4 percent of the initial amount owed each month—obviously, if you keep using the credit card, it will take longer.

Concept Check

1. Why does the future value of a given amount increase when interest is compounded nonannually as opposed to annually?
2. How do you adjust the present- and future-value formulas when interest is compounded monthly?

The Present Value of an Uneven Stream and Perpetuities

Although some projects will involve a single cash flow and some will involve annuities, many projects will involve uneven cash flows over several years. Chapter 10, which examines investments in fixed assets, presents this situation repeatedly. There we will be comparing not only the present value of cash flows generated by different projects but also the cash inflows and outflows generated by a particular project to determine that project's present value. However, this will not be difficult because the present value of any cash flow is measured in today's dollars and thus can be compared, through addition for inflows and subtraction for outflows, to the present value of any other cash flow also measured in today's dollars. For example, if we wished to find the present value of the following cash flows

YEAR	CASH FLOW	YEAR	CASH FLOW
1	\$ 0	6	500
2	200	7	500
3	−400	8	500
4	500	9	500
5	500	10	500

given a 6 percent discount rate, we would merely discount the flows back to the present and total them by adding in the positive flows and subtracting the negative ones. However, this problem is complicated by the annuity of \$500 that runs from years 4 through 10. To accommodate this, we can first discount the annuity back to the beginning of period 4 (or end of period 3) and get its present value at that point in time. We then bring this single cash flow (which is the present value of the 7-year annuity) back to the present. In effect, we discount twice, first back to the end of period 3, then back to the present. This is shown graphically in Table 5-5 and numerically in Table 5-6. Thus, the present value of this uneven stream of cash flows is \$2,185.69.

TABLE 5-5 The Present Value of an Uneven Stream Involving One Annuity Discounted to the Present at 6 Percent: An Example

YEAR	$r = 6\%$										
	0	1	2	3	4	5	6	7	8	9	10
Dollars received at end of year		0	200	-400	500	500	500	500	500	500	500
	\$ 178.00										
	-335.85										
				\$2,791.19							
	2,343.54										
Total present value	<u>\$2,185.69</u>										

4 Determine the present value of an uneven stream of payments and understand perpetuities.

STEP 1

Bring the \$200 at the end of year 2 back to present:

CALCULATOR SOLUTION

Data Input	Function Key
2	N
6	I/Y
200	FV
0	PMT
Function Key	Answer

CPT**PV** −178.00**STEP 2**

Bring the negative \$400 at the end of year 3 back to present:

CALCULATOR SOLUTION

Data Input	Function Key
3	N
6	I/Y
0	PMT
−400	FV
Function Key	Answer

CPT**PV** 335.85**STEP 3**

Bring the 7-year, \$500 annuity beginning at the end of year 4 back to the beginning of year 4, which is the same as the end of year 3:

CALCULATOR SOLUTION

Data Input	Function Key
7	N
6	I/Y
500	PMT
0	FV
Function Key	Answer

CPT**PV** −2,791.19

STEP 4

Bring the value we just calculated, which is the value of the 7-year annuity of \$500 at the end of year 3, back to present.

CALCULATOR SOLUTION

Data Input	Function Key
3	N
6	I/Y
2,791.19	FV
0	PMT
Function Key	Answer
CPT	
PV	-2,343.54

STEP 5

Add the present value of the cash inflows and subtract the present value of the cash outflow (the \$400 from the end of year 3) calculated in steps 1, 2, and 4.

\$ 178.00
 -335.85
 +2,343.54
\$ 2,185.69

perpetuity an annuity with an infinite life.

TABLE 5-6 Determining the Present Value of an Uneven Stream Involving One Annuity Discounted to the Present at 6 Percent: An Example

1. Present value of \$200 received at the end of 2 years at 6% =	\$ 178.00
2. Present value of a \$400 outflow at the end of 3 years at 6% =	-335.85
3. (a) Value at end of year 3 of a \$500 annuity, years 4–10 at 6% = \$2,791.19	
(b) Present value of \$2,791.19 received at the end of year 3 at 6% =	<u>2,343.54</u>
4. Total present value =	\$2,185.69

Remember, once the cash flows from an investment have been brought back to the present, they can be combined by adding and subtracting to determine the project's total present value.

Concept Check

- How would you calculate the present value of an investment that produced cash flows of \$100 received at the end of year 1 and \$700 at the end of year 2?

Perpetuities

A **perpetuity** is *an annuity that continues forever*; that is, every year following its establishment this investment pays the same dollar amount. An example of a perpetuity is preferred stock that pays a constant dollar dividend infinitely. Determining the present value of a perpetuity is delightfully simple; we merely need to divide the constant flow by the discount rate. For example, the present value of a \$100 perpetuity discounted back to the present at 5 percent is $\$100/0.05 = \$2,000$. Thus, the equation representing the present value of a perpetuity is

$$PV = \frac{PP}{r} \quad (5-9)$$

where: PV = the present value of the perpetuity

PP = the constant dollar amount provided by the perpetuity

r = the annual interest (or discount) rate

EXAMPLE 5.13

Calculating the future value of an investment

What is the present value of a \$500 perpetuity discounted back to the present at 8 percent?

STEP 1: FORMULATE A SOLUTION STRATEGY

The present value of the perpetuity can be calculated using equation (5-9) as follows:

$$PV = \frac{PP}{r}$$

STEP 2: CRUNCH THE NUMBERS

Substituting $PP = \$500$ and $r = 0.08$ into equation (5-9), we find

$$PV = \frac{\$500}{0.08} = \$6,250$$

STEP 3: ANALYZE YOUR RESULTS

Thus, the present value of this perpetuity is \$6,250.

Concept Check

1. What is a perpetuity?
2. When r , the annual interest (or discount) rate, increases, what happens to the present value of a perpetuity? Why?

TABLE 5-7 Summary of Time Value of Money Equations

Calculation	Equation
Future value of a single payment	$FV_n = PV(1 + r)^n$
Present value of a single payment	$PV = FV_n \left[\frac{1}{(1 + r)^n} \right]$
Future value of an annuity	$FV \text{ of an annuity} = PMT \left[\frac{(1 + r)^n - 1}{r} \right]$
Present value of an annuity	$PV \text{ of an annuity} = PMT \left[\frac{1 - (1 + r)^{-n}}{r} \right]$
Future value of an annuity due	$FV_n(\text{annuity due}) = \text{future value of an annuity} \times (1 + r)$
Present value of an annuity due	$PV(\text{annuity due}) = \text{present value of an annuity} \times (1 + r)$
Effective annual return (EAR) =	$\left[1 + \frac{\text{quoted rate}}{m} \right]^m - 1$
Future value of a single payment with nonannual compounding	$FV_n = PV \left(1 + \frac{r}{m} \right)^{mn}$
Present value of a perpetuity	$PV = \frac{PP}{r}$
Notations: FV_n = the future value of the investment at the end of n years n = the number of years until payment will be received or during which compounding occurs r = the annual interest or discount rate PV = the present value of the future sum of money m = the number of times compounding occurs during the year PMT = the annuity payment deposited or received at the end of each year PP = the constant dollar amount provided by the perpetuity	

Chapter Summaries

Explain the mechanics of compounding and bringing the value of money back to the present. (pgs. 143–157)



SUMMARY: Compound interest occurs when interest paid on an investment during the first period is added to the principal; then, during the second period, interest is earned on this new sum. The formula for this appears in Table 5-7.

Although there are several ways to move money through time, they all give you the same result. In the business world, the primary method is through the use of a financial spreadsheet, with Excel being the most popular. If you can use a financial calculator, you can easily apply your skills to a spreadsheet.

Actually, we have only one formula with which to calculate both present value and future value—we simply solve for different variables— FV and PV . This single compounding formula is $FV_n = PV(1 + r)^n$ and is presented in Table 5-7.

KEY TERMS

Compound interest, page 144 The situation in which interest paid on an investment during the first period is added to the principal. During the second period, interest is earned on the original principal plus the interest earned during the first period.

Future value, page 145 The amount to which your investment will grow, or a future dollar amount.

Future value factor, page 145 The value of $(1 + r)^n$ used as a multiple to calculate an amount's future value.

Simple interest, page 146 If you only earned interest on your initial investment, it would be referred to as simple interest.

Present value, page 153 The value in today's dollars of a future payment discounted back to present at the required rate of return.

Present value factor, page 153 The value of $\frac{1}{(1 + r)^n}$ used as a multiplier to calculate an amount's present value.

KEY EQUATIONS

Future value at the end of year $n = \text{present value} \times (1 + r)^n$

Present value = future value at the end of year $n \times \frac{1}{(1 + r)^n}$

**Understand annuities.** (pgs. 157–164)

SUMMARY: An annuity is a series of equal payments made for a specified number of years. In effect, it is calculated as the sum of the present or future value of the individual cash flows over the life of the annuity. The formula for an annuity due is given in Table 5-7.

If the cash flows from an annuity occur at the end of each period, the annuity is referred to as an ordinary annuity. If the cash flows occur at the beginning of each period, the annuity is referred to as an annuity due. We will assume that cash flows occur at the end of each period unless otherwise stated.

The procedure for solving for PMT, the annuity payment value when i , n , and PV are known, is also used to determine what payments are associated with paying off a loan in equal installments over time. Loans that are paid off this way, in equal periodic payments, are called amortized loans. Although the payments are fixed, different amounts of each payment are applied toward the principal and the interest. With each payment you make, you owe a bit less on the principal. As a result, the amount that goes toward the interest payment declines with each payment made, whereas the portion of each payment that goes toward the principal increases.

KEY TERMS

Annuity, page 157 A series of equal dollar payments made for a specified number of years.

Ordinary annuity, page 157 An annuity where the cash flows occur at the end of each period.

Compound annuity, page 157 Depositing an equal sum of money at the end of each year for a certain number of years and allowing it to grow.

Annuity future value factor, page 158 The value of $\left[\frac{(1 + r)^n - 1}{r} \right]$ used as a multiplier to calculate the future value of an annuity.

Annuity present value factor, page 160 The value of $\left[\frac{1 - (1 + r)^{-n}}{r} \right]$ used as a multiplier to calculate the present value of an annuity.

Annuity due, page 161 An annuity in which the payments occur at the beginning of each period.

Amortized loan, page 162 A loan that is paid off in equal periodic payments.

KEY EQUATIONS

$$\text{Future value of an annuity} = PMT \left[\frac{(1 + r)^n - 1}{r} \right]$$

$$\text{Present value of an annuity} = PMT \left[\frac{1 - \frac{1}{(1 + r)^n}}{r} \right]$$

$$\text{Future value of an annuity due} = PMT \left[\frac{(1 + r)^n - 1}{r} \right] \times (1 + r)$$

$$\text{Present value of an annuity due} = PMT \left[\frac{1 - \frac{1}{(1 + r)^n}}{r} \right] \times (1 + r)$$

Determine the future or present value of a sum when there are nonannual compounding periods. (pgs. 165–169)


SUMMARY: With nonannual compounding, interest is earned on interest more frequently because the length of the compounding period is shorter. As a result, there is an inverse relationship between the length of the compounding period and the effective annual interest rate. The formula for solving for the future value of a single payment with nonannual compounding is given in Table 5-7.

KEY TERM

Effective annual rate (EAR), page 165 The annual compound rate that produces the same return as the nominal, or quoted, rate when

something is compounded on a nonannual basis. In effect, the EAR provides the true rate of return.

KEY EQUATIONS

$$\text{Effective annual rate (EAR)} = \left(1 + \frac{\text{quoted rate}}{m} \right)^m - 1$$

$$FV_n = PV \left[1 + \frac{r}{m} \right]^{m \cdot n}$$

Determine the present value of an uneven stream of payments and understand perpetuities. (pgs. 169–171)


SUMMARY: Although some projects will involve a single cash flow and some will involve annuities, many projects will involve uneven cash flows over several years. However, finding the present or future value of these flows is not difficult because the present value of any cash flow measured in today's dollars can be compared, by adding the inflows and subtracting the outflows, to the present value of any other cash flow also measured in today's dollars.

A perpetuity is an annuity that continues forever; that is, every year following its establishment the investment pays the same dollar amount. An example of a perpetuity is preferred stock, which pays a constant dollar dividend infinitely. Determining the present value of a perpetuity is delightfully simple. We merely need to divide the constant flow by the discount rate.

KEY TERM

Perpetuity, page 170 An annuity with an infinite life.

KEY EQUATION

$$\text{Present value of a perpetuity} = \frac{\text{constant dollar amount provided by the perpetuity}}{r}$$

Review Questions

All Review Questions are available in [MyFinanceLab](#).

- 5-1. What is the time value of money? Why is it so important?
- 5-2. The process of discounting and compounding are related. Explain this relationship.
- 5-3. How would an increase in the interest rate (r) or a decrease in the holding period (n) affect the future value (FV_n) of a sum of money? Explain why.
- 5-4. Suppose you were considering depositing your savings in one of three banks, all of which pay 5 percent interest; bank A compounds annually, bank B compounds semiannually, and bank C compounds daily. Which bank would you choose? Why?
- 5-5. What is an annuity? Give some examples of annuities. Distinguish between an annuity and a perpetuity.
- 5-6. Compare some of the different financial calculators that are available on the Internet. Look at www.dinkytown.net, www.bankrate.com/calculators.aspx, and www.interest.com/calculators. Which financial calculators do you find to be the most useful? Why?

Study Problems

All Study Problems are available in [MyFinanceLab](#).



- 5-1. (Compound interest) To what amount will the following investments accumulate?
 - a. \$5,000 invested for 10 years at 10 percent compounded annually
 - b. \$8,000 invested for 7 years at 8 percent compounded annually
 - c. \$775 invested for 12 years at 12 percent compounded annually
 - d. \$21,000 invested for 5 years at 5 percent compounded annually
- 5-2. (Compound value solving for n) How many years will the following take?
 - a. \$500 to grow to \$1,039.50 if invested at 5 percent compounded annually
 - b. \$35 to grow to \$53.87 if invested at 9 percent compounded annually
 - c. \$100 to grow to \$298.60 if invested at 20 percent compounded annually
 - d. \$53 to grow to \$78.76 if invested at 2 percent compounded annually
- 5-3. (Compound value solving for r) At what annual rate would the following have to be invested?
 - a. \$500 to grow to \$1,948.00 in 12 years
 - b. \$300 to grow to \$422.10 in 7 years
 - c. \$50 to grow to \$280.20 in 20 years
 - d. \$200 to grow to \$497.60 in 5 years
- 5-4. (Present value) What is the present value of the following future amounts?
 - a. \$800 to be received 10 years from now discounted back to the present at 10 percent
 - b. \$300 to be received 5 years from now discounted back to the present at 5 percent
 - c. \$1,000 to be received 8 years from now discounted back to the present at 3 percent
 - d. \$1,000 to be received 8 years from now discounted back to the present at 20 percent
- 5-5. (Compound value) Stanford Simmons, who recently sold his Porsche, placed \$10,000 in a savings account paying annual compound interest of 6 percent.
 - a. Calculate the amount of money that will have accrued if he leaves the money in the bank for 1, 5, and 15 years.
 - b. If he moves his money into an account that pays 8 percent or one that pays 10 percent, rework part (a) using these new interest rates.
 - c. What conclusions can you draw about the relationship between interest rates, time, and future sums from the calculations you have completed in this problem?
- 5-6. (Future value) Sarah Wiggum would like to make a single investment and have \$2 million at the time of her retirement in 35 years. She has found a mutual fund that will earn 4 percent annually. How much will Sarah have to invest today? What if Sarah were a finance major and learned how to earn a 14 percent annual return, how much would she have to invest today?
- 5-7. (Future value) Sales of a new finance book were 15,000 copies this year and were expected to increase by 20 percent per year. What are expected sales during each of the next 3 years? Graph this sales trend and explain.

5-8. (Future value) Albert Pujols hit 47 home runs in 2009. If his home-run output grew at a rate of 12 percent per year, what would it have been over the following 5 years?

5-9. (Solving for r in compound interest) If you were offered \$1,079.50 10 years from now in return for an investment of \$500 currently, what annual rate of interest would you earn if you took the offer?

5-10. (Solving for r in compound interest) You lend a friend \$10,000, for which your friend will repay you \$27,027 at the end of 5 years. What interest rate are you charging your “friend”?

5-11. (Present-value comparison) You are offered \$1,000 today, \$10,000 in 12 years, or \$25,000 in 25 years. Assuming that you can earn 11 percent on your money, which should you choose?

5-12. (Solving for r in compound interest—financial calculator needed) In September 1963, the first issue of the comic book *X-MEN* was issued. The original price for that issue was \$0.12. By September 2013, 50 years later, the value of the near-mint copy of this comic book had risen to \$33,000. What annual rate of interest would you have earned if you had bought the comic in 1963 and sold it in 2013?

5-13. (Compounding using a calculator) Bart Simpson, age 10, wants to be able to buy a really cool new car when he turns 16. His really cool car costs \$15,000 today, and its cost is expected to increase 3 percent annually. Bart wants to make one deposit today (he can sell his mint-condition original *Nuclear Boy* comic book) into an account paying 7.5 percent annually in order to buy his car in 6 years. How much will Bart’s car cost, and how much does Bart have to save today in order to buy this car at age 16?

5-14. (Compounding using a calculator) Lisa Simpson wants to have \$1 million in 45 years by making equal annual end-of-the-year deposits into a tax-deferred account paying 8.75 percent annually. What must Lisa’s annual deposit be?

5-15. (Future value) Bob Terwilliger received \$12,345 for his services as financial consultant to the mayor’s office of his hometown of Springfield. Bob says that his consulting work was his civic duty and that he should not receive any compensation. So, he has invested his paycheck into an account paying 3.98 percent annual interest and left the account in his will to the city of Springfield on the condition that the city could not collect any money from the account for 200 years. How much money will the city receive from Bob’s generosity in 200 years?

5-16. (Solving for r) Kirk Van Houten, who has been married for 23 years, would like to buy his wife an expensive diamond ring with a platinum setting on their 30-year wedding anniversary. Assume that the cost of the ring will be \$12,000 in 7 years. Kirk currently has \$4,510 to invest. What annual rate of return must Kirk earn on his investment to accumulate enough money to pay for the ring?

5-17. (Solving for n) Jack asked Jill to marry him, and she has accepted under one condition: Jack must buy her a new \$330,000 Rolls-Royce Phantom. Jack currently has \$45,530 that he may invest. He has found a mutual fund that pays 4.5% annual interest in which he will place the money. How long will it take Jack to win Jill’s hand in marriage?

5-18. (Present value) Ronen Consulting has just realized an accounting error which has resulted in an unfunded liability of \$398,930 due in 28 years. Toni Flanders, the company’s CEO, is scrambling to discount the liability to the present to assist in valuing the firm’s stock. If the appropriate discount rate is 7 percent, what is the present value of the liability?

5-19. (Future value) Selma and Patty Bouvier are twins and both work at the Springfield DMV. Selma and Patty Bouvier decide to save for retirement, which is 35 years away. They’ll both receive an 8 percent annual return on their investment over the next 35 years. Selma invests \$2,000 per year at the end of each year *only* for the first 10 years of the 35-year period—for a total of \$20,000 saved. Patty doesn’t start saving for 10 years and then saves \$2,000 per year at the end of each year for the remaining 25 years—for a total of \$50,000 saved. How much will each of them have when they retire?

5-20. (Compound annuity) What is the accumulated sum of each of the following streams of payments?

- \$500 a year for 10 years compounded annually at 5 percent
- \$100 a year for 5 years compounded annually at 10 percent
- \$35 a year for 7 years compounded annually at 7 percent
- \$25 a year for 3 years compounded annually at 2 percent

5-21. (Present value of an annuity) What is the present value of the following annuities?

- \$2,500 a year for 10 years discounted back to the present at 7 percent
- \$70 a year for 3 years discounted back to the present at 3 percent



- c. \$280 a year for 7 years discounted back to the present at 6 percent
- d. \$500 a year for 10 years discounted back to the present at 10 percent

5-22. (Solving for r with annuities) Nicki Johnson, a sophomore mechanical engineering student, receives a call from an insurance agent, who believes that Nicki is an older woman ready to retire from teaching. He talks to her about several annuities that she could buy that would guarantee her an annual fixed income. The annuities are as follows:

ANNUITY	INITIAL PAYMENT INTO ANNUITY (AT $t = 0$)	AMOUNT OF MONEY RECEIVED PER YEAR	DURATION OF ANNUITY (YEARS)
A	\$50,000	\$8,500	12
B	\$60,000	\$7,000	25
C	\$70,000	\$8,000	20

If Nicki could earn 11 percent on her money by placing it in a savings account, should she place it instead in any of the annuities? Which ones, if any? Why?

5-23. (Loan amortization) Mr. Bill S. Preston, Esq., purchased a new house for \$80,000. He paid \$20,000 down and agreed to pay the rest over the next 25 years in 25 equal end-of-year payments plus 9 percent compound interest on the unpaid balance. What will these equal payments be?

5-24. (Solving for PMT of an annuity) To pay for your child's education, you wish to have accumulated \$15,000 at the end of 15 years. To do this you plan on depositing an equal amount into the bank at the end of each year. If the bank is willing to pay 6 percent compounded annually, how much must you deposit each year to reach your goal?

5-25. (Future value of an annuity) In 10 years you are planning on retiring and buying a house in Oviedo, Florida. The house you are looking at currently costs \$100,000 and is expected to increase in value each year at a rate of 5 percent. Assuming you can earn 10 percent annually on your investments, how much must you invest at the end of each of the next 10 years to be able to buy your dream home when you retire?

5-26. (Compound value) The Aggarwal Corporation needs to save \$10 million to retire a \$10 million mortgage that matures in 10 years. To retire this mortgage, the company plans to put a fixed amount into an account at the end of each year for 10 years, with the first payment occurring at the end of 1 year. The Aggarwal Corporation expects to earn 9 percent annually on the money in this account. What equal annual contribution must it make to this account to accumulate the \$10 million in 10 years?

5-27. (Loan amortization) On December 31, Beth Klemkosky bought a yacht for \$50,000, paying \$10,000 down and agreeing to pay the balance in 10 equal end-of-year installments and 10 percent interest on the declining balance. How big would the annual payments be?

5-28. (Solving for r of an annuity) You lend a friend \$30,000, which your friend will repay in five equal annual end-of-year payments of \$10,000, with the first payment to be received 1 year from now. What rate of return does your loan receive?

5-29. (Loan amortization) A firm borrows \$25,000 from the bank at 12 percent compounded annually to purchase some new machinery. This loan is to be repaid in equal installments at the end of each year over the next 5 years. How much will each annual payment be?

5-30. (Compound annuity) You plan on buying some property in Florida 5 years from today. To do this you estimate that you will need \$20,000 at that time for the purchase. You would like to accumulate these funds by making equal annual deposits in your savings account, which pays 12 percent annually. If you make your first deposit at the end of this year, and you would like your account to reach \$20,000 when the final deposit is made, what will be the amount of your deposits?

5-31. (Loan amortization) On December 31, Son-Nan Chen borrowed \$100,000, agreeing to repay this sum in 20 equal end-of-year installments and 15 percent interest on the declining balance. How large must the annual payments be?

5-32. (Loan amortization) To buy a new house you must borrow \$150,000. To do this you take out a \$150,000, 30-year, 10 percent mortgage. Your mortgage payments, which are made at the end of each year (one payment each year), include both principal and 10 percent interest on the declining balance. How large will your annual payments be?

5-33. (Spreadsheet problem) If you invest \$900 in a bank in which it will earn 8 percent compounded annually, how much will it be worth at the end of 7 years? Use a spreadsheet to do your calculations.

5-34. (Spreadsheet problem) In 20 years you'd like to have \$250,000 to buy a vacation home, but you have only \$30,000. At what rate must your \$30,000 be compounded annually for it to grow to \$250,000 in 20 years? Use a spreadsheet to calculate your answer.

5-35. (Compounding using a calculator and annuities due) Springfield mogul Montgomery Burns, age 80, wants to retire at age 100 in order to steal candy from babies full time. Once Mr. Burns retires, he wants to withdraw \$1 billion at the beginning of each year for 10 years from a special offshore account that will pay 20 percent annually. In order to fund his retirement, Mr. Burns will make 20 equal end-of-the-year deposits in this same special account that will pay 20 percent annually. How much money will Mr. Burns need at age 100, and how large of an annual deposit must he make to fund this retirement account?

5-36. (Compounding using a calculator and annuities due) Imagine Homer Simpson actually invested \$100,000 5 years ago at a 7.5 percent annual interest rate. If he invests an additional \$1,500 a year at the beginning of each year for 20 years at the same 7.5 percent annual rate, how much money will Homer have 20 years from now?

5-37. (Solving for r in an annuity) Your folks just called and would like some advice from you. An insurance agent just called them and offered them the opportunity to purchase an annuity for \$21,074.25 that will pay them \$3,000 per year for 20 years, but they don't have the slightest idea what return they would be making on their investment of \$21,074.25. What rate of return will they be earning?

5-38. (Compound interest with nonannual periods)

- Calculate the future sum of \$5,000, given that it will be held in the bank 5 years at an annual interest rate of 6 percent.
- Recalculate part (a) using compounding periods that are (1) semiannual and (2) bimonthly.
- Recalculate parts (a) and (b) for a 12 percent annual interest rate.
- Recalculate part (a) using a time horizon of 12 years (annual interest rate is still 6 percent).
- With respect to the effect of changes in the stated interest rate and holding periods on future sums in parts (c) and (d), what conclusions do you draw when you compare these figures with the answers found in parts (a) and (b)?



5-39. (Compound interest with nonannual periods) After examining the various personal loan rates available to you, you find that you can borrow funds from a finance company at 12 percent compounded monthly or from a bank at 13 percent compounded annually. Which alternative is more attractive?

5-40. (Solving for n with nonannual periods) About how many years would it take for your investment to grow fourfold if it were invested at 16 percent compounded semiannually?

5-41. (Spreadsheet problem) To buy a new house you take out a 25-year mortgage for \$300,000. What will your monthly payments be if the interest rate on your mortgage is 8 percent? Use a spreadsheet to calculate your answer. Now, calculate the portion of the 48th monthly payment that goes toward interest and principal.

5-42. (Nonannual compounding using a calculator) Prof. Finance is thinking about trading cars. He estimates he will still have to borrow \$25,000 to pay for his new car. How large will Prof. Finance's monthly car loan payment be if he can get a 5-year (60 equal monthly payments) car loan from the university's credit union at 6.2 percent?

5-43. (Nonannual compounding using a calculator) Bowflex's television ads say you can get a fitness machine that sells for \$999 for \$33 a month for 36 months. What rate of interest are you paying on this Bowflex loan?

5-44. (Nonannual compounding using a calculator) Ford's current incentives include 4.9 percent financing for 60 months or \$1,000 cash back for a Mustang. Let's assume Suzie Student wants to buy the premium Mustang convertible, which costs \$25,000, and she has no down payment other than the cash back from Ford. If she chooses the \$1,000 cash back, Suzie can borrow from the VTech Credit Union at 6.9 percent for 60 months (Suzie's credit isn't as good as Prof. Finance's). What will Suzie Student's monthly payment be under each option? Which option should she choose?

5-45. (*Nonannual compounding using a calculator*) Ronnie Rental plans to invest \$1,000 at the end of each quarter for 4 years into an account that pays 6.4 percent compounded quarterly. He will use this money as a down payment on a new home at the end of the 4 years. How large will his down payment be 4 years from today?

5-46. (*Nonannual compounding using a calculator*) Dennis Rodman has a \$5,000 debt balance on his Visa card that charges 12.9 percent compounded monthly. Dennis's current minimum monthly payment is 3 percent of his debt balance, which is \$150. How many months (round up) will it take Dennis to pay off his credit card if he pays the current minimum payment of \$150 at the end of each month?

5-47. (*Nonannual compounding using a calculator*) Should we have bet the kids' college fund at the dog track? Let's look at one specific case of a college professor (let's call him Prof. ME) with two young children. Two years ago, Prof. ME invested \$160,000 hoping to have \$420,000 available 12 years later when the first child started college. However, the account's balance is now only \$140,000. Let's figure out what is needed to get Prof. ME's college savings plan back on track.

- What was the original annual rate of return needed to reach Prof. ME's goal when he started the fund 2 years ago?
- Now with only \$140,000 in the fund and 10 years remaining until his first child starts college, what annual rate of return would the fund have to earn to reach Prof. ME's \$420,000 goal if he adds nothing to the account?
- Shocked by his experience of the past 2 years, Prof. ME feels the college mutual fund has invested too much in stocks. He wants a low-risk fund in order to ensure he has the necessary \$420,000 in 10 years, and he is willing to make end-of-the-month deposits to the fund as well. He later finds a fund that promises to pay a guaranteed return of 6 percent compounded monthly. Prof. ME decides to transfer the \$140,000 to this new fund and make the necessary monthly deposits. How large of a monthly deposit must Prof. ME make into this new fund to meet his \$420,000 goal?
- Now Prof. ME gets sticker shock from the necessary monthly deposit he has to make into the guaranteed fund in the preceding question. He decides to invest the \$140,000 today and \$500 at the end of each month for the next 10 years into a fund consisting of 50 percent stock and 50 percent bonds and hope for the best. What annual rate of return would the fund have to earn in order to reach Prof. ME's \$420,000 goal?



5-48. (*Present value of an uneven stream of payments*) You are given three investment alternatives to analyze. The cash flows from these three investments are as follows:

END OF YEAR	INVESTMENT		
	A	B	C
1	\$10,000		\$10,000
2	10,000		
3	10,000		
4	10,000		
5	10,000	\$10,000	
6		10,000	50,000
7		10,000	
8		10,000	
9		10,000	
10		10,000	10,000

Assuming a 20 percent discount rate, find the present value of each investment.

5-49. (*Present value*) The Kumar Corporation is planning on issuing bonds that pay no interest but can be converted into \$1,000 at maturity, 7 years from their purchase. To price these bonds competitively with other bonds of equal risk, it is determined that they should yield 10 percent, compounded annually. At what price should the Kumar Corporation sell these bonds?

5-50. (Perpetuities) What is the present value of the following?

- A \$300 perpetuity discounted back to the present at 8 percent
- A \$1,000 perpetuity discounted back to the present at 12 percent
- A \$100 perpetuity discounted back to the present at 9 percent
- A \$95 perpetuity discounted back to the present at 5 percent

5-51. (Complex present value) How much do you have to deposit today so that beginning 11 years from now you can withdraw \$10,000 a year for the next 5 years (periods 11 through 15) plus an *additional* amount of \$20,000 in that last year (period 15)? Assume an interest rate of 6 percent.

5-52. (Complex present value) You would like to have \$50,000 in 15 years. To accumulate this amount you plan to deposit each year an equal sum in the bank, which will earn 7 percent interest compounded annually. Your first payment will be made at the end of the year.

- How much must you deposit annually to accumulate this amount?
- If you decide to make a large lump-sum deposit today instead of the annual deposits, how large should this lump-sum deposit be? (Assume you can earn 7 percent on this deposit.)
- At the end of 5 years you will receive \$10,000 and deposit this in the bank toward your goal of \$50,000 at the end of 15 years. In addition to this deposit, how much must you deposit in equal annual deposits to reach your goal? (Again assume you can earn 7 percent on this deposit.)

5-53. (Comprehensive present value) You are trying to plan for retirement in 10 years, and currently you have \$100,000 in a savings account and \$300,000 in stocks. In addition you plan on adding to your savings by depositing \$10,000 per year in your savings account at the end of each of the next 5 years and then \$20,000 per year at the end of each year for the final 5 years until retirement.

- Assuming your savings account returns 7 percent compounded annually, and your investment in stocks will return 12 percent compounded annually, how much will you have at the end of 10 years? (Ignore taxes.)
- If you expect to live for 20 years after you retire, and at retirement you deposit all of your savings in a bank account paying 10 percent, how much can you withdraw each year after retirement (20 equal withdrawals beginning 1 year after you retire) to end up with a zero balance upon your death?

5-54. (Present value) The state lottery's million-dollar payout provides for \$1 million to be paid over 19 years in 20 payments of \$50,000. The first \$50,000 payment is made immediately, and the 19 remaining \$50,000 payments occur at the end of each of the next 19 years. If 10 percent is the appropriate discount rate, what is the present value of this stream of cash flows? If 20 percent is the appropriate discount rate, what is the present value of the cash flows?

5-55. (Complex annuity) Upon graduating from college 35 years ago, Dr. Nick Riviera was already thinking of retirement. Since then he has made deposits into his retirement fund on a quarterly basis in the amount of \$300. Nick has just completed his final payment and is at last ready to retire. His retirement fund has earned 9% interest compounded quarterly.

- How much has Nick accumulated in his retirement account?
- In addition to all this, 15 years ago, Nick received an inheritance check for \$20,000 from his beloved uncle. He decided to deposit the entire amount into his retirement fund. What is his current balance in the fund?

5-56. (Complex annuity and future value) Milhouse, 22, is about to begin his career as a rocket scientist for a NASA contractor. Being a rocket scientist, Milhouse knows that he should begin saving for retirement immediately. Part of his inspiration came from reading an article on Social Security in *Newsweek*, where he saw that the ratio of workers paying taxes to retirees collecting checks will drop dramatically in the future. In fact, the ratio was 8.6 workers for every retiree in 1955; today the ratio is 3.3, and it will drop to 2 workers for every retiree in 2040. Milhouse's retirement plan pays 9 percent interest annually and allows him to make equal yearly contributions. Upon retirement Milhouse plans to buy a new boat, which he estimates will cost him \$300,000 in 43 years (he plans to retire at age 65). He also estimates that in order to live comfortably he will require a yearly income of \$80,000 for each year after he retires. Based on family history, Milhouse expects to live until age 80 (that is, he would like to receive 15 payments of \$80,000 at the end of each year). When he retires, Milhouse will purchase his boat in one lump sum and place the remaining balance into an account, which pays 6% interest, from which he will withdraw his \$80,000 per year. If Milhouse's first contribution is made

1 year from today, and his last is made the day he retires, how much money must he contribute each year to his retirement fund?

5-57. (Present value of a complex stream) Don Draper has signed a contract that will pay him \$80,000 at the end of each year for the next 6 years, plus an additional \$100,000 at the end of year 6. If 8 percent is the appropriate discount rate, what is the present value of this contract?

5-58. (Present value of a complex stream) Don Draper has signed a contract that will pay him \$80,000 at the *beginning* of each year for the next 6 years, plus an additional \$100,000 at the end of year 6. If 8 percent is the appropriate discount rate, what is the present value of this contract?

5-59. (Complex stream of cash flows) Roger Sterling has decided to buy an ad agency and is going to finance the purchase with seller financing—that is, a loan from the current owners of the agency. The loan will be for \$2,000,000 financed at a 7 percent nominal annual interest rate. This loan will be paid off over 5 years with end-of-month payments along with a \$500,000 balloon payment at the end of year 5. That is, the \$2 million loan will be paid off with monthly payments and there will also be a final payment of \$500,000 at the end of the final month. How much will the monthly payments be?

5-60. (Future and present value using a calculator) In 2013 Bill Gates was worth about \$28 billion after he reduced his stake in Microsoft from 21 percent to around 14 percent by moving billions into his charitable foundation. Let's see what Bill Gates can do with his money in the following problems.

- I'll take Manhattan? Manhattan's native tribe sold Manhattan Island to Peter Minuit for \$24 in 1626. Now, 387 years later in 2013, Bill Gates wants to buy the island from the "current natives." How much would Bill have to pay for Manhattan if the "current natives" want a 6 percent annual return on the original \$24 purchase price? Could he afford it?
- (*Nonannual compounding using a calculator*) How much would Bill have to pay for Manhattan if the "current natives" want a 6% return compounded monthly on the original \$24 purchase price?
- Microsoft Seattle? Bill Gates decides to pass on Manhattan and instead plans to buy the city of Seattle, Washington, for \$60 billion in 10 years. How much would Mr. Gates have to invest today at 10 percent compounded annually in order to purchase Seattle in 10 years?
- Now assume Bill Gates wants to invest only about half his net worth today, \$14 billion, in order to buy Seattle for \$60 billion in 10 years. What annual rate of return would he have to earn in order to complete his purchase in 10 years?
- Margaritaville? Instead of buying and running large cities, Bill Gates is considering quitting the rigors of the business world and retiring to work on his golf game. To fund his retirement, Bill Gates would invest his \$28 billion fortune in safe investments with an expected annual rate of return of 7 percent. Also, Mr. Gates wants to make 40 equal annual withdrawals from this retirement fund beginning a year from today. How much can Mr. Gates's annual withdrawal be in this case?

Mini Case

This Mini Case is available in [MyFinanceLab](#).

For your job as the business reporter for a local newspaper, you are given the task of putting together a series of articles that explain the power of the time value of money to your readers. Your editor would like you to address several specific questions in addition to demonstrating for the readership the use of time value of money techniques by applying them to several problems. What would be your response to the following memorandum from your editor?

To: Business Reporter
 From: Perry White, Editor, *Daily Planet*
 Re: Upcoming Series on the Importance and Power of the Time Value of Money

In your upcoming series on the time value of money, I would like to make sure you cover several specific points. In addition, before you begin this assignment, I want to make sure we are all reading from the same script, as accuracy has always been the cornerstone of the *Daily Planet*. In this regard, I'd like a response to the following questions before we proceed:

- What is the relationship between discounting and compounding?
- What is the relationship between the present-value factor and the annuity present-value factor?

- c.
 - 1. What will \$5,000 invested for 10 years at 8 percent compounded annually grow to?
 - 2. How many years will it take \$400 to grow to \$1,671 if it is invested at 10 percent compounded annually?
 - 3. At what rate would \$1,000 have to be invested to grow to \$4,046 in 10 years?
- d. Calculate the future sum of \$1,000, given that it will be held in the bank for 5 years and earn 10 percent compounded semiannually.
- e. What is an annuity due? How does this differ from an ordinary annuity?
- f. What is the present value of an ordinary annuity of \$1,000 per year for 7 years discounted back to the present at 10 percent? What would be the present value if it were an annuity due?
- g. What is the future value of an ordinary annuity of \$1,000 per year for 7 years compounded at 10 percent? What would be the future value if it were an annuity due?
- h. You have just borrowed \$100,000, and you agree to pay it back over the next 25 years in 25 equal end-of-year payments plus 10 percent compound interest on the unpaid balance. What will be the size of these payments?
- i. What is the present value of a \$1,000 perpetuity discounted back to the present at 8 percent?
- j. What is the present value of a \$1,000 annuity for 10 years, with the first payment occurring at the end of year 10 (that is, ten \$1,000 payments occurring at the end of year 10 through year 19), given a discount rate of 10 percent?
- k. Given a 10 percent discount rate, what is the present value of a perpetuity of \$1,000 per year if the first payment does not begin until the end of year 10?